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DESIGN OF COMPOSITE MATERIAL STRUCTURES FOR BUCKLING  
- AN EVALUATION OF THE STATE-OF-THE-ART



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20. The Appendix contains an annotated bibliography, listing 95 references on the subject.

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## FOREWORD

This report was prepared by Lockheed Missiles and Space Company, Inc., Palo Alto Research Laboratories, 3251 Hanover Street, Palo Alto, California, in partial fulfillment of the requirements under Contract F33615-76-C-3105. The effort was initiated under Project 2307, "Research in Flight Vehicle Structures," Task 2307N102, "Research in the Behavior of Metallic and Composite Components of Air Frame Structures." The project monitor for the contract was Dr Narendra S. Khot of the Structures and Dynamics Division (AFWAL/FIBRA).

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## TABLE OF CONTENTS

<u>Section</u>		<u>Page</u>
	LIST OF ILLUSTRATIONS	vi
I	SUMMARY	1
II	INTRODUCTION	2
III	THE GENERAL THEORY	4
IV	TRANSVERSE SHEAR EFFECTS	7
V	IMPERFECTION SENSITIVITY AND POSTBUCKLING BEHAVIOR	9
VI	EXPERIMENTAL VERIFICATION	13
VII	REFERENCES	18
APPENDIX		
	BIBLIOGRAPHY ON BUCKLING OF COMPOSITE STRUCTURES	
	A. GENERAL THEORY	A-1
	B. EFFECTS OF TRANSVERSE SHEAR	A-11
	C. IMPERFECTION SENSITIVITY AND POSTBUCKLING STRENGTH	A-15
	D. EXPERIMENTAL VERIFICATION	A-21

## LIST OF ILLUSTRATIONS

<u>Figure</u>	<u>Title</u>	<u>Page</u>
1	Buckling loads for cross-ply square plate, $\nu_{12} = 0.25$ , $G_{12}/E_{22} = 0.5$ .	20
2	Directions of transverse shear stresses.	21
3	Error due to omission of shear effects in long isotropic plates (all edges simply supported).	22
4	Error due to omission of shear effects in square laminated plates (all edges simply supported).	23
5	Different types of load-displacement relations.	24
6	Empirical factor $\phi$ for cylinders subjected to axial compression.	25
7	Effect of initial imperfection amplitude ( $w_0$ ) on the buckling load of composite cylinder.	26
8	Effect of initial imperfection amplitude $w_0$ on the buckling load of composite cylinder.	27
9	Strain reversal at buckling.	28
10	Load-deflection diagram and Southwell plot for simply supported composite plate (from Reference 18, Test 203b).	29

## Section I

### SUMMARY

Composite materials have favorable stiffness/weight and strength/weight ratios. Since the technology is relatively new and use of such materials introduces special problems, there has been some reluctance among designers to adopt composites in primary structures, particularly for buckling critical components. The purpose of the review of the literature presented in this report is to illustrate the special problems connected with use of composites. The body of the report summarizes the conclusions the author felt could be based on the available material.

The Appendix contains an annotated bibliography, listing 95 references on the subject.



## Section II

### INTRODUCTION

The use of composite materials offers many advantages. Most important among these appear to be the favorable stiffness/weight and strength/weight ratios and the possibilities to "tailor" the material for specific purposes. However, the technology is relatively new and there seems to be some apprehension among designers about the use of such materials. When it comes to use of composites in buckling critical compression members, the questions raised include the following:

- o Is the basic theory for buckling of shells and plates valid for materials that are heterogeneous and display a coupling between bending and membrane action?
- o In view of the low modulus of the matrix, is it generally necessary to include effects of transverse shear in computation of buckling loads?
- o In view of the brittleness of the material, will it remain competitive in situations where metal structures show considerable post-buckling strength?
- o What are the effects of certain peculiarities in composite material behavior such as:
  - . nonlinear stress-strain curves
  - . different moduli in tension and compression
  - . sensitivity to damage and stress concentrations
- o In comparison to metal structures, are those made from a composite material more or less sensitive to geometric deviations from the

true form? What are the effects of other types of imperfections, such as:

- . initial delaminations
- . nonuniform distributions of fibers within the laminae

In the following, the state of the art in design of plates and shells of composite material is summarized and the need for additional research in the area is discussed. Comments on some publications in this general area are included in the appended bibliography.

### SECTION III THE GENERAL THEORY

Most engineering materials such as metals are for all practical purposes homogeneous. Composite materials, on the other hand, are heterogeneous in the respect that the material properties vary with location. This heterogeneity has prompted the question of the applicability of the general theory of shells. In a first-order shell theory it is assumed that the strains at any point within the shell (or plate) can be expressed in terms of strains and curvature changes at some reference surface. By introduction of the constitutive relations for the material and integration through the shell thickness, force and moment resultants are obtained as functions of the reference surface deformation. For stiffness determination, it appears that each of the laminae in a laminated shell wall can be assumed to be orthotropic and homogeneous. Stiffness properties for the different lamina in the shell wall can be determined experimentally or from the properties of the constituent materials. Practical experience indicates that the experimental approach is the more accurate. Once the properties of the laminae are known, the integration through the thickness is readily carried out. Typical for the composite material is that all the coefficients  $C_{ij}$  are nonzero in the relation

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_{12} \\ M_1 \\ M_2 \\ M_{12} \end{Bmatrix} = [C] \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_{12} \\ \kappa_1 \\ \kappa_2 \\ \kappa_{12} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_{12} \\ \kappa_1 \\ \kappa_2 \\ \kappa_{12} \end{Bmatrix} \quad (1)$$

where  $A$ ,  $B$  and  $D$  are  $3 \times 3$  submatrices.

In theory, nonlinear and stability equations derived on the basis of Equations (1) represent the shell behavior just as well for composite materials as for metal shells. The accuracy of results obtained depends on the accuracy in the coefficients of the  $C$ -matrix. In general, experiments on the stiffness properties of the laminate indicate that these coefficients can be determined with reasonable accuracy.

The most important restriction in a first order shell theory is that it neglects the effects of transverse shear. Since this is a problem of special importance for composite materials, the effects of transverse shear will be dealt with separately in the next section.

A special problem with use of composite materials in plates and shells is that with as many as 21 independent parameters in the constitutive equations, it becomes virtually impossible to give design charts for buckling of plates and shells. These parameters are sensitive to fiber orientation and stacking sequences. Many of the publications discussing buckling of composite structures based on a first order shell theory are concerned with the effects of nonzero elements in the  $B$ -matrix (Equations 1), that is, with the effects of coupling between membrane and bending action. Other papers consider optimum choice of fiber orientation and stacking sequence and special problems such as the effects of different moduli in tension and compression or of a nonlinear stress-strain curve in shear.

Within its scope, the classical theory of stability of plates and shells is no less satisfactory for composite material than it is for isotropic material. A special feature of the composite material behavior is that the bifurcation buckling load depends on a large number of independent "shell wall parameters". In particular, the parameters introducing coupling between membrane and bending behavior can have a profound effect on the critical load. However, for shell walls with a reasonably large number of layers, this effect is reduced. Figure 1 (from Reference 1) shows how the critical load varies with the orientation of the fibers for angle-ply with 2 and 6 plies, respectively.

In Reference 2, an approximate method is presented to determine the buckling load for shells with membrane bending coupling. The coupling terms are neglected in the constitutive Equations (1), but instead the terms in the D-matrix are altered to reflect the coupling effect.

Use of the "reduced bending stiffness" matrix,  $D^*$ , given by

$$D^* = D - B A^{-1} B \quad (2)$$

reduces the problem to an "equivalent" problem with no membrane-bending coupling present. The method has been evaluated in many subsequent analyses with the consensus that it gives a good approximation. However, Reference 3 finds this to be true only if principal directions in all laminae are parallel to plate or shell boundaries and Reference 4 warns that it may be inaccurate.

Computer programs are available for computation of buckling loads for plates and shells with all terms included in the shell wall stiffness matrix. Such programs are certainly available to practically any organization involved in the design and manufacture of composite material structures. Computer programs are available also that, for a given plate or shell configuration and intensity of loading, compute the optimum fiber orientation and required shell thickness. Such programs may eliminate the trial and error procedure in initial design that is best carried out by use of design charts and simple approximate formulas.

Charts for a quick preliminary sizing of a plate or shell structure are still desired by many designers. Despite the large number of parameters, it may be possible to produce such charts if they are restricted to one layup, chosen to be optimum for a particular type of application. For cylindrical shells under axial compression, it would be possible to produce charts giving the critical load as a function of shell thickness, radius and length. The charts would be valid only if sufficiently many layers are included so the coupling effects can be eliminated, and then only for say an equal mix of fibers with  $0^\circ$ ,  $\pm 45^\circ$ ,  $90^\circ$  orientation.

#### Section IV

#### TRANSVERSE SHEAR EFFECTS

Plate and shell theories represent means to simplify the analysis by introduction of assumptions that make the displacements functions of two rather than three spatial coordinates. Usually this reduction is achieved by use of the assumptions:

- o Normals to the reference surface remain straight during deformation
- o Normals to the reference surface remain normal after deformation
- o The transverse normal stress is negligibly small

The assumption that the normals remain normal to the deformed surface means that transverse shear deformation can be neglected. Such an assumption is certainly acceptable if the shell is sufficiently thin. In the following, such theories are referred to as first order theories. A second order theory may, for example, be obtained if the first of the three assumptions is retained but the second discarded. Such theories have been presented by Reissner and Mindlin (References 5, 6). Higher order theories can be obtained if also the first assumption is discarded, but it seems questionable whether use of such theories would have any advantages in comparison to a complete three-dimensional analysis.

In keeping with generally accepted nomenclature, we will refer to longitudinal and transverse elastic moduli ( $E_1$  and  $E_2$ ) as the moduli in the plane of the lamina parallel and normal to the fiber direction. The inplane shear modulus,  $G_{12}$ , is distinguished from the two transverse shear moduli,  $G_{13}$  and  $G_{23}$ . The modulus  $G_{13}$  corresponding to shear across the fibers (compare Figure 2) is generally somewhat larger than  $G_{23}$ . Typical values for a lamina may be of the order  $G_{13} = 0.6 E_2$  and  $G_{23} = 0.4 E_2$ . Since the transverse shear moduli for

a lamina are small in comparison to the longitudinal elastic modulus, the transverse shear deformations must have a bigger effect on the buckling load of composites than it has on metal plates or shells. That is, the composite shells must be thinner relative to inplane dimensions or wavelengths before the transverse shear effect can be omitted.

Since most of the available computer programs presently do not include the effects of transverse shear, it is important to the designer to know the limits of the first order theory. For an isotropic material like a metal plate, results presented in Reference 7 indicate that  $b/h$  for a simply supported plate may be close to 10 before the error exceeds 5% (See Figure 3). In order to obtain similar accuracy for composite material, we must restrict the first order theory to even thinner plates. Results from Reference 8 (See Figure 4) indicate that for a material with  $E_1/E_2 = 30$ , the transverse shear effects should be included if the width-to-thickness ratio is less than about 20. It should be noted that both the references quoted above were applied to plates with simply supported edges. With respect to buckling of flat plates, the effect of clamping the edges is essentially equivalent to reducing the inplane dimensions by a factor of two. It might be surmised then that for clamped plates the transverse shear effect should be accounted for if  $b/h < 40$ . Similarly, it appears that for a flange with one free edge the effect can be omitted if  $b/h > 10$ . It seems desirable that a few more numerical results be made available for guidance of design. In particular, if composites are used at elevated hygrothermal conditions,  $E_1/E_2$  may be very large.

## Section V

### IMPERFECTION SENSITIVITY AND POSTBUCKLING BEHAVIOR

In the previous section, the discussion has been restricted to the analysis of buckling loads according to the classical theory of bifurcation buckling. This theory is based on an investigation of the stability of the equilibrium on the primary loading path. That is, the path corresponding to a load displacement curve through the origin. However, knowledge about the classical buckling load may not always be sufficient for design purposes and the classical theory has often been supplemented by nonlinear analysis and special theories to determine imperfection sensitivity.

The classical bifurcation buckling load represents the lowest load level at which the primary path is intersected by a loading path corresponding to a secondary equilibrium form. Since the form of equilibrium corresponding to the primary path loses its stability at the bifurcation point, the structural behavior must be governed by the behavior on the secondary path. If the equilibrium on this path is stable, the structure can take on additional load, while the deformation pattern (the buckling mode) on the secondary path gradually increases. On the other hand, if the equilibrium on the secondary path is unstable, there will be no stable equilibrium form in the immediate neighborhood



of the bifurcation point. In that case, the consequence is a violent transition (snap-through) to another equilibrium form. Actually, the presence of a shape imperfection will cause deformations containing a component of the buckling mode at any load level and the bifurcation point will actually never be reached.

The two cases discussed above are illustrated in Figure 5. The first of these, representing stable equilibrium on the secondary path, is typical for the behavior of flat plates. The second, representing an unstable secondary path, is typical of the behavior of many shells. In particular, the axially loaded cylindrical shell, as being extremely imperfection sensitive, has been the subject of many studies.

The problem of imperfection sensitivity affecting the buckling of cylindrical shells was recognized earlier, but the classical paper on the subject is Koiter's thesis (Reference 9). Koiter's results have been summarized and discussed in simpler terms in later publications such as Reference 10. The basic Koiter theory evaluates the stability of the equilibrium on the secondary path. For the case in which only a single buckling mode is associated with the bifurcation point, the secondary equilibrium path can be expressed in the form

$$\lambda = \frac{P}{P_{CL}} = 1 + a \delta + b \delta^2 + \dots \quad (3)$$

where  $P_{CL}$  is the bifurcation buckling load,  $\delta$  is the amplitude of the buckling mode. The equilibrium is stable only if  $\lambda > 1$  in the limit  $\delta \rightarrow 0$ . Since  $\delta$  can have any sign, the necessary requirement for stable equilibrium is that  $a = 0$ . In that case, the sign of the constant  $b$  determines the stability. For the cylindrical shell under axial compression,  $a = 0$  and  $b < 0$ .

Koiter also presents a simple approximate formula for the critical load as a function of the imperfection sensitivity parameter constants ( $a$  or  $b$ ) and the amplitude of an imperfection in the form of the buckling mode. This approach is generally referred to as Koiter's general theory.

If the imperfections for a shell of revolution are axisymmetric, the prebuckling deformation mode will still not contain any component of the nonsymmetric buckling modes. Consequently, bifurcation into a nonsymmetric buckling mode remains a possibility for such shells. It is shown in Reference 11 that the critical load for

axially loaded cylinders is very sensitive to axially symmetric imperfections. The analysis of cylinders with purely axisymmetric imperfections is sometimes referred to as Koiter's special theory.

When composite materials are used in plate and shell structures, the following questions arise:

- 1) Metal plates with supported edges can carry loads several times the bifurcation buckling load. In view of the brittleness of the composite materials, will these remain weight-efficient in such applications?
- 2) Are cylinders of composite material (with anisotropy and membrane-bending coupling) more or less imperfection sensitive than the isotropic cylinders?
- 3) Will the actual size of the geometric imperfection in practical applications be more or less severe than they are in metal cylinders?
- 4) Are other types of imperfections affecting the buckling load for composite material structures, such as voids and delaminations?

It has long been known that shell structures, in particular axially loaded cylinders, collapse or snap through at loads well below the classical buckling load. The presence of small initial deviations from the ideal geometry have been recognized as the reason for this discrepancy. In design of such shells, an empirical knockdown factor is applied to the critical load. The knockdown factor varies with the thickness of the shell and is generally given in terms of the parameter  $R/h$  as shown in Figure 6 (from Reference 12).

A number of publications present an attempt to answer the question of whether cylindrical shells of composite material are more or less sensitive to small geometric imperfections than are the isotropic cylinders. A nonlinear analysis (Reference 13), Koiter's general theory (Reference 14), and Koiter's special theory (Reference 15) indicate that, in comparison to isotropic cylinders,

composite cylinders may be somewhat less sensitive to geometric imperfections. Also, it is clear that cylinders with close to optimum fiber orientation are most sensitive. This is illustrated by Figures 7 and 8.

The possibility remains that composite cylinders, while less sensitive to imperfections, as manufactured display more severe imperfections and possibly more severe knockdown factors. Additional observations of experiments and measurements on practical structures are required before this question can be satisfactorily answered. The possibility must be faced that the composite material plates and shells contain flaws of other types than those that affect isotropic cylinders. It does not seem likely that delaminations will pass undetected through any reasonable inspection if they are large enough to cause the type of separate buckling that is discussed in Reference 16. However, smaller delaminations will still reduce the stiffness of the shell and, although the results of the bulk of test data seems to be reassuring, the fear that repeated loading can cause a growth of such flaws is not completely dispelled.

## Section VI

### EXPERIMENTAL VERIFICATION

A final evaluation of the state of the art must, of course, be based on results from laboratory tests and from the experience acquired by use of composite material in structural applications in the past.

Many experimental results on the buckling of composite material plates and shells have been presented over the last few years. In general, they tend to indicate that the theory for composites is approximately on a par with the theory for metal shells with respect to its reliability. It may be prudent, however, to observe that quality control may be better for the laboratory test specimens than for mass-produced structural components. The literature scanned during this study contains little information about the performance of actual hardware.

For structures with a stable postbuckling equilibrium path, it may be difficult to determine the actual buckling load from the observations made during the test. For this reason, many experimenters are using the so-called Southwell plot in order to determine the critical load,  $P_{CR}$ . The Southwell plot was first applied in the evaluation of column buckling tests. It is based on the

fact that the tested column is not absolutely straight. Let the amplitude in the first harmonic in a Fourier series representing the geometric interpretation be equal to  $a$ . The lateral displacement  $\delta$  under an axial load  $P$  (see Reference 10) then is

$$\delta = \frac{\alpha}{1-\alpha} a \sin \frac{\pi x}{\ell} \quad (4)$$

where

$$\alpha = P/P_{CR}$$

From this equation we find

$$\frac{P_{CR}}{P} = \frac{\delta+a}{\delta} \quad \text{or} \quad (5)$$

$$\frac{\delta}{P} = \frac{1}{P_{CR}} (a+\delta) = \frac{a}{P_{CR}} + \frac{1}{P_{CR}} \delta$$

In application of the Southwell plot  $P$  and  $\delta$  are measured. A trace of  $\frac{\delta}{P}$  versus  $\delta$  should emerge as a straight line. The inverse of the critical load is given by the slope of this line.

This procedure works very well if the buckling loads for the different modes are well separated as they are for the column and in many cases for flat plates. The only way to obtain disagreement between the classical buckling load for a column and the experimental load determined by the Southwell method is to misjudge boundary conditions or column stiffness  $EI$ . It seems that the Southwell plot may be useful to establish actual boundary conditions for example.

If the eccentricity caused by the coupling or by initial deviations from flatness is small enough, a sharp change of direction can be observed in the load displacement curve. The bifurcation buckling load can then be directly established from experimental results. In that case, it represents the load level at which significant lateral displacement begins to develop and is thus a meaningful design parameter. If the imperfection is so large that a buckling load cannot be established by direct observation of lateral displacement or strain reversal, then

the bifurcation buckling point is not a meaningful design parameter (see Figure 9). For plates of composite material, coupling between membrane and bending effects often gives rise to relatively large lateral deformations below the buckling load. In that case, only a nonlinear analysis gives useful information about structural behavior, the relative unimportance of the bifurcation point makes efforts to use Southwell plots or other means to find a correlation between test and bifurcation theory rather meaningless.

For shells such as complete cylinders, the membrane-bending coupling does not cause deformations that resemble the critical buckling pattern. These shells are usually imperfection sensitive and the development of buckling mode is usually not noticed before failure occurs. In such cases, the Southwell plot has been used with some success for nondestructive testing. Such procedures would be of special interest in connection with use of composites because the quality control problem is acute.

While there is no doubt that the classical bifurcation buckling load can be accurately computed, the question of its significance as a design parameter remains. The results presented in Reference 17 are based on test specimens manufactured from cloth. In that case, the membrane-bending coupling terms vanish. Presumably, initial geometric deviations are small and a strain reversal is clearly observed when the buckling load is reached. On the other hand, tests on unsymmetric laminates reported in Reference 18 ( $0^\circ$ ,  $90^\circ$ ,  $0$ ,  $90^\circ$ ) show a gradual growth of the lateral displacement from the onset of loading. The curves in Figure 10, showing the lateral displacement versus load and a Southwell plot, indicate that the classical buckling load (and consequently the Southwell plot) has little significance.

Since flat plates with supported edges in shear as well as under normal loading generally can carry loads above the buckling load, failure does not occur at the buckling load in either case. The ability to carry load is limited by the load level at which fracture occurs or possibly when allowable deformation is exceeded. Consequently, design of flat plates is not properly based on the bifurcation buckling analysis, but rather on a nonlinear postbuckling analysis.

It appears then that flat plates in compression or shear fail at a load level corresponding to rather large strains and the question arises whether the advantages of the composite materials in such applications may be eliminated by

their relative brittleness (lower strain at failure). This question is not clearly answered by the research results presented so far. Clearly, sufficiently thin plates can be loaded well into the postbuckling range in shear. Reference 19 reports that shear panels with  $b/h$  of about 300 carried loads of at least 8 times the classical buckling loads. Some panels were cycled to five times the bifurcation load 100 times. Test results reported in Reference 20 on buckling of shear plates with  $b/h$  of 100 show ultimate loads between two and three times the classical buckling load. A nonlinear analysis combined with Tsai's fracture criterion (Reference 21) predicts reasonably well the ultimate load of these panels.

For thicker plates ( $b/h > 20$ ) loaded in compression, Reference 22 indicates failure as low as 60% to 70% of the classical buckling load. In view of the observation above, it does not seem likely that this discrepancy may be caused by transverse shear. More likely, it is a result of the bending strains caused by initial geometric imperfections. The problem of failure of flat plates under compression or shear loading requires some additional study.

In Reference 11, a procedure is presented based on Koiter's special theory and the assumption that cylinders with the same effective  $R/h$  have identical amplitudes of the axisymmetric imperfections. Since the growth of an imperfection pattern is governed by the radius of gyration of the shell wall, the effective thickness is defined in Reference 11 in terms of the coefficients in the  $C$ -matrix such that it is proportional to this ratio and equals the thickness for the isotropic cylinder. The results of the predictions of the method were compared to the few test results available at the time. All test specimens failed above the prediction by the method, but a comparison shows that it is only slightly less conservative than direct application of the knockdown factor for the isotropic cylinder with the same effective  $R/h$ . A similar evaluation of results obtained in later experimental investigations would be of value. There seems to be little reason to recommend different knockdown factors for cylinders of composite materials under axial compression than those chosen from charts for the isotropic cylinder (see Figure 7, for example). For cylinders in torsion or external pressure, a knockdown factor of about 0.8 seems to be appropriate (see Reference 23).

For cylindrical panels (or local buckling between stiffeners for complete cylinders) the results in Reference 24 indicate that similar knockdown factors must be used. Of course, for narrower panels, the situation is more favorable. Sufficiently narrow panels of isotropic material are able to carry loads above the critical load. Due to brittleness, such must be further explored before it can be recommended for composite material panels.

It must be noted that in a few experiments extremely low buckling loads were obtained and the results were discarded as not representative. It is possible that similar results in some cases went unreported. While the test results generally support the use of composite materials in stability critical structural components, some doubts remain and additional research is advisable. In particular, this is true when it comes to the use of composite material at elevated hygrothermal conditions. The effects of the viscoelastic nature of the matrix in such applications appears to be essentially unexplored. Such special problems as nonlinear stress-strain curve for shear or different tension and compression moduli seem to have little effect on the critical load. However, more research should be devoted to the effects of transverse shear and certainly additional experimental results are most welcome. Finally, the development of methods for nondestructive testing, possibly based on the Southwell plot (see Reference 24, for example), may eventually allow the designer of composite shells to sleep well at night.



## Section VII

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- [10] Brush, D. O., and Almroth, B. O., *Buckling of Bars, Plates, and Shells*, McGraw-Hill, 1975.
- [11] Koiter, W. T., The Effect of Axisymmetric Imperfections on the Buckling of Cylindrical Shells Under Axial Compression, *Proc. K. Ned. Akad. Wet.*, Amsterdam, Ser. B., Vol. 6, 1963; also. Lockheed Missiles and Space Company, Rept. 6-90-63-86, August 1963.
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- [18] Mandell, J. F., An Experimental Investigation of Anisotropic Fiber Reinforced Plastic Plates, AFML TR-68-281.
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- [20] Booton, M., Buckling of Imperfect Anisotropic Cylinders Under Combined Loading, Institute for Aerospace Studies, Univ. of Toronto, UTIAS Rept. No. 203, August 1976.
- [21] Tsai, S. W., Strength Characteristics of Composite Materials, NASA OR-224, April 1965.
- [22] Williams, J. G., and Mikulas, Jr., M. M., Analytical and Experimental Study of Structurally Efficient Composite Hat-Stiffened Panels Loaded in Axial Compression, AIAA/ASME/SAE 16th SDM Meeting, Denver, Colorado, May 1975.
- [23] Tennyson, R. C., Buckling of Laminated Composite Cylinders: A Review, J. Composite Matls., Vol. 11, January 1975, pp. 17-24.
- [24] Wilkins, D. J., Compression Buckling Tests of Laminated Graphite-Epoxy Curved Panels, AIAA, 12th Aerospace Sci. Meeting, Washington, D.C., 30 January-1 February 1974.

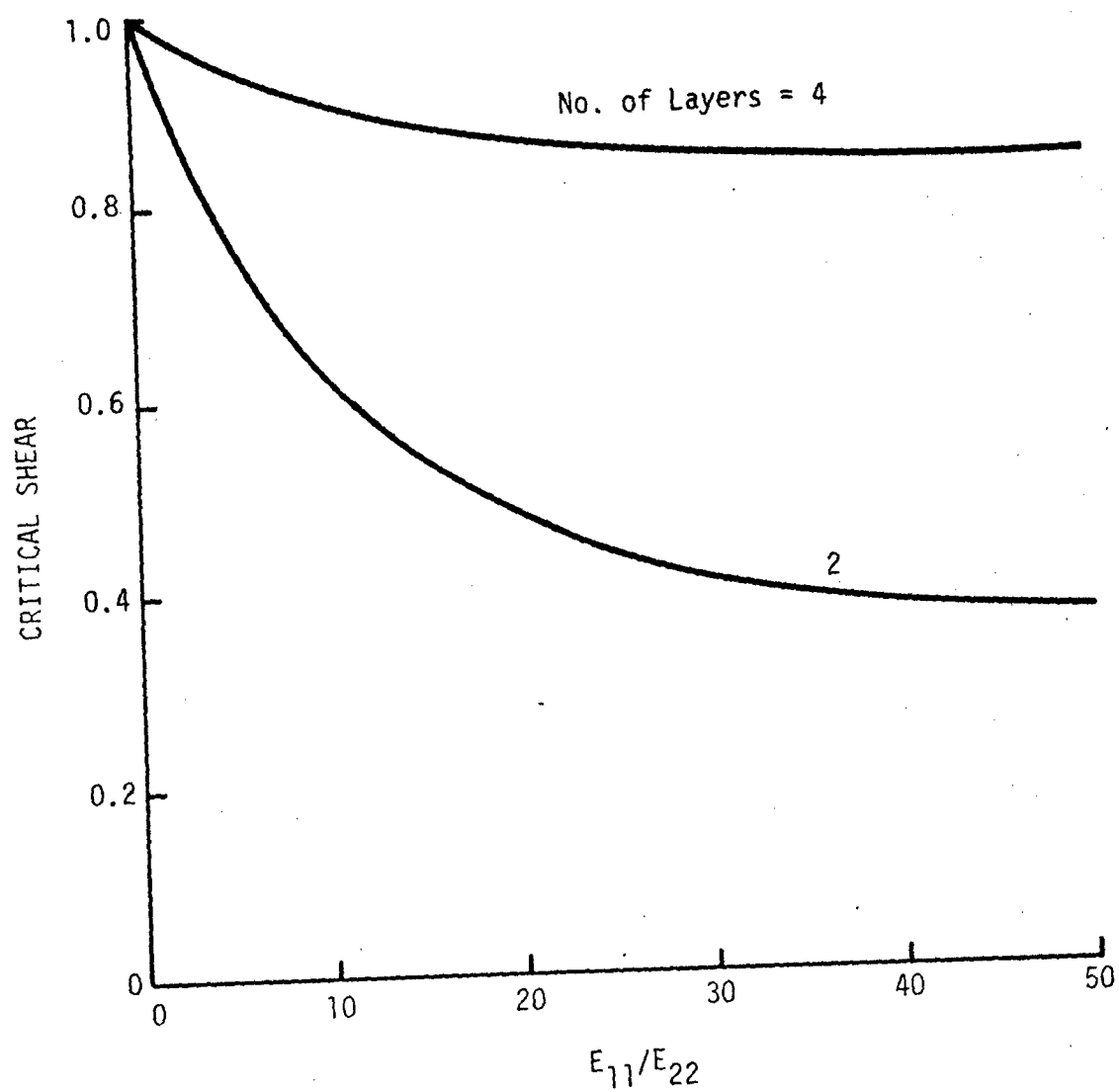


Figure 1 Buckling loads for cross-ply square plate,  $\nu_{12} = 0.25$ ,  
 $G_{12}/E_{22} = 0.5$ .

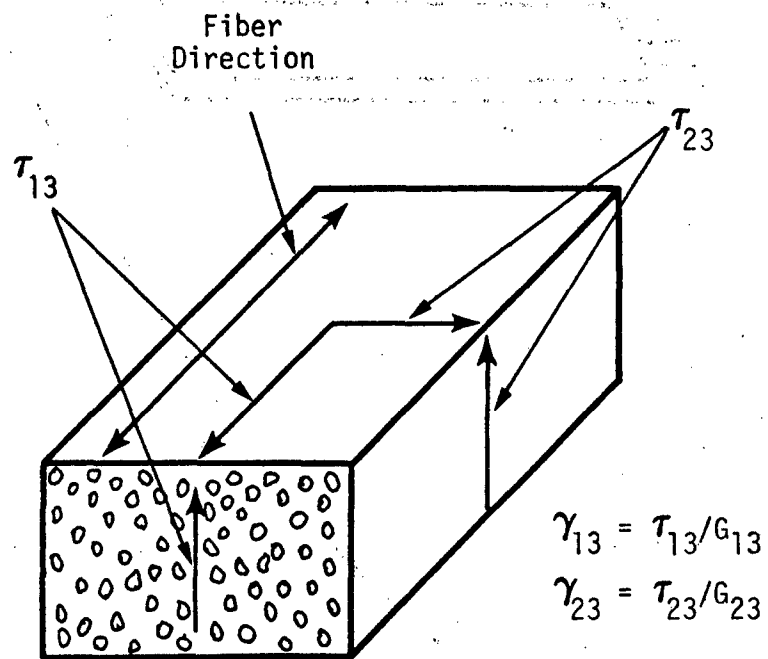


Figure 2 Directions of transverse shear stresses.

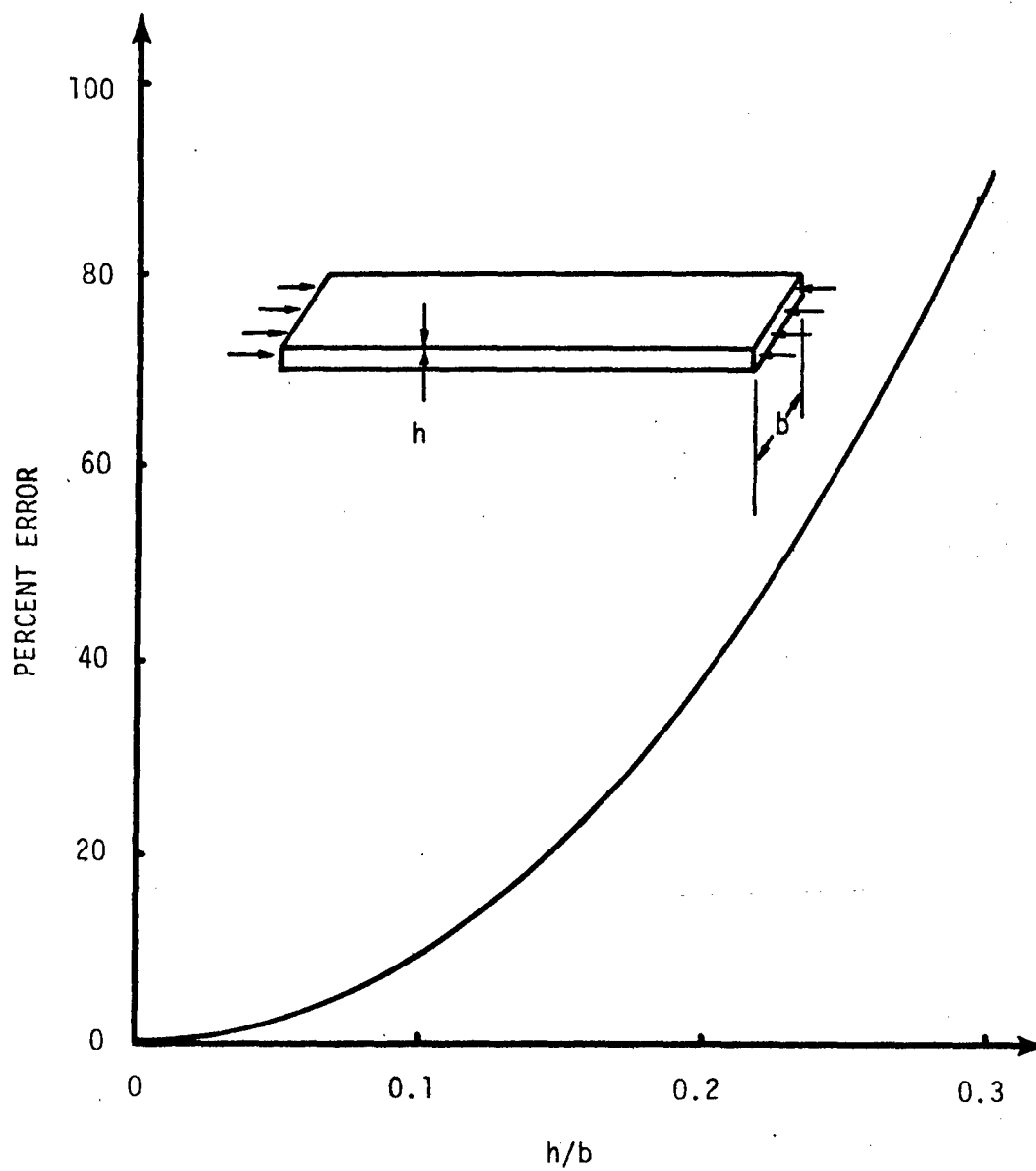
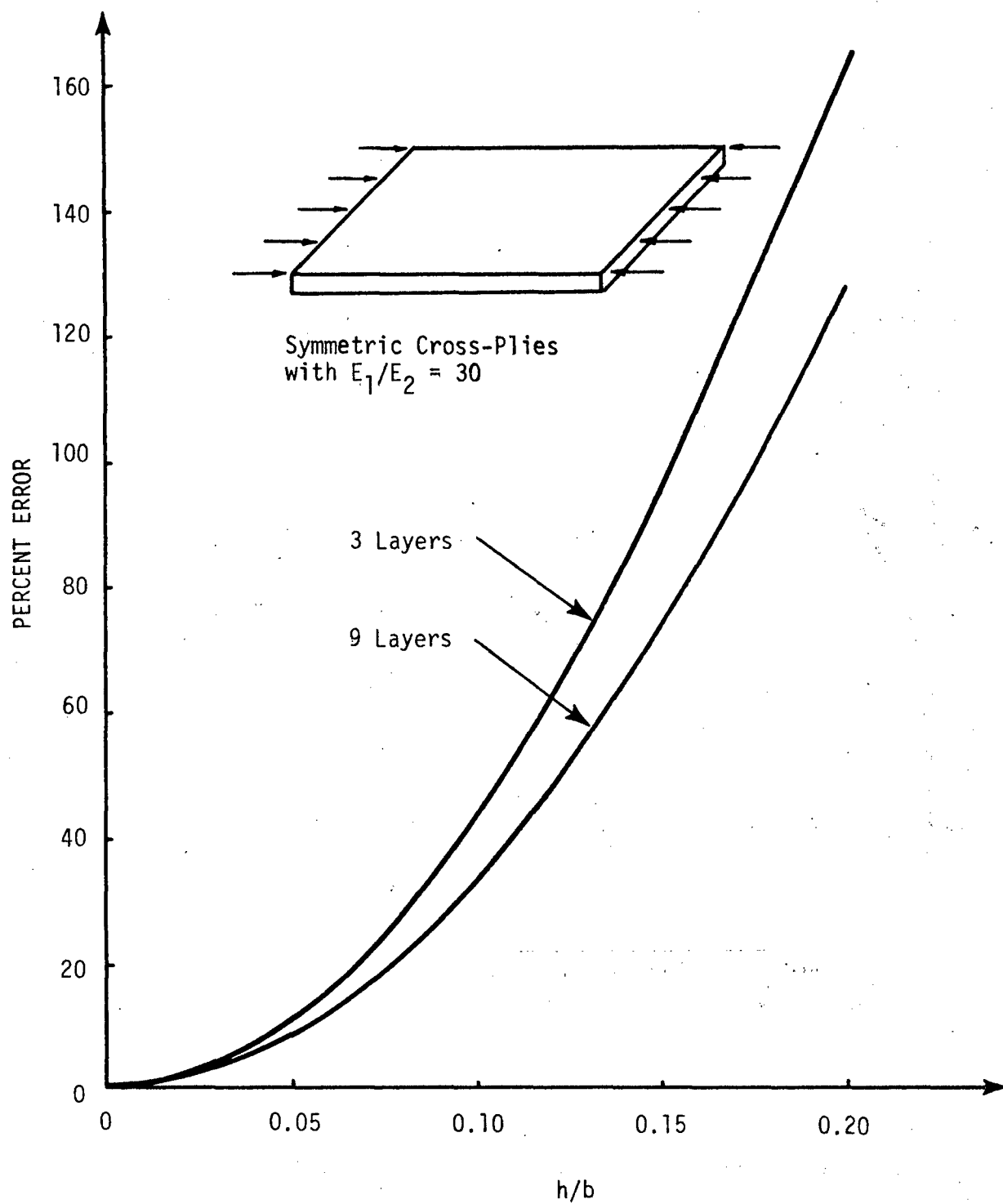
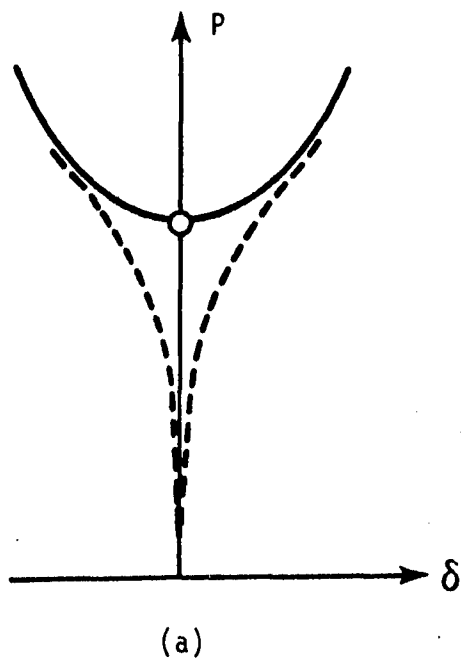


Figure 3 Error due to omission of shear effects in long isotropic plates (all edges simply supported).

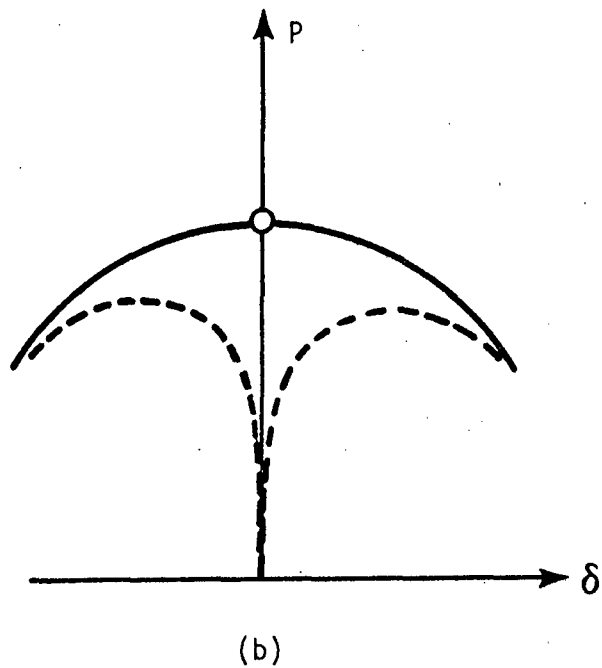


**Figure 4** Error due to omission of shear effects in square laminated plates (all edges simply supported).

STABLE POSTBUCKLING



UNSTABLE POSTBUCKLING



————— PERFECT STRUCTURE  
----- IMPERFECT STRUCTURE

Figure 5 Different types of load-displacement relations.

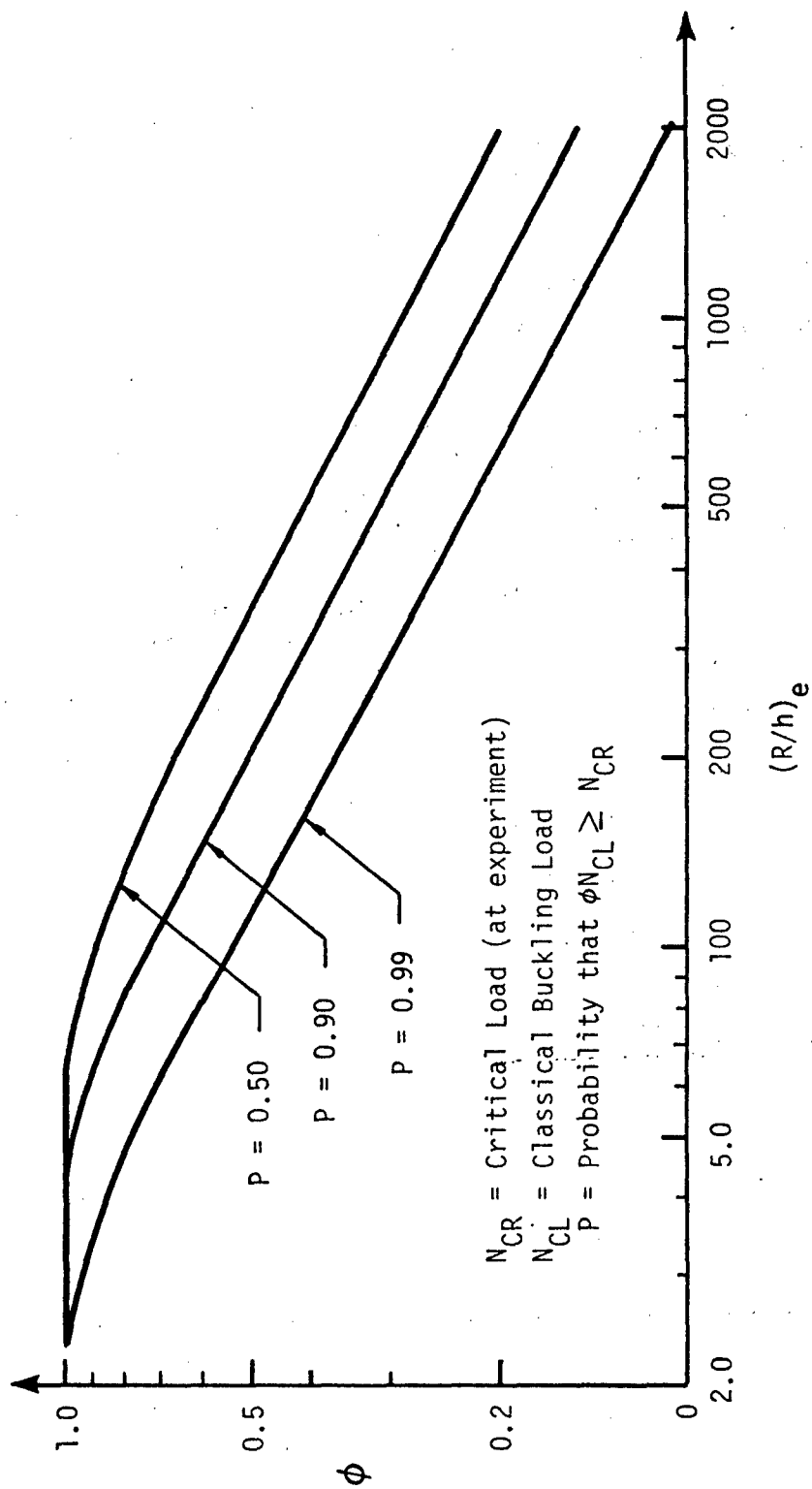
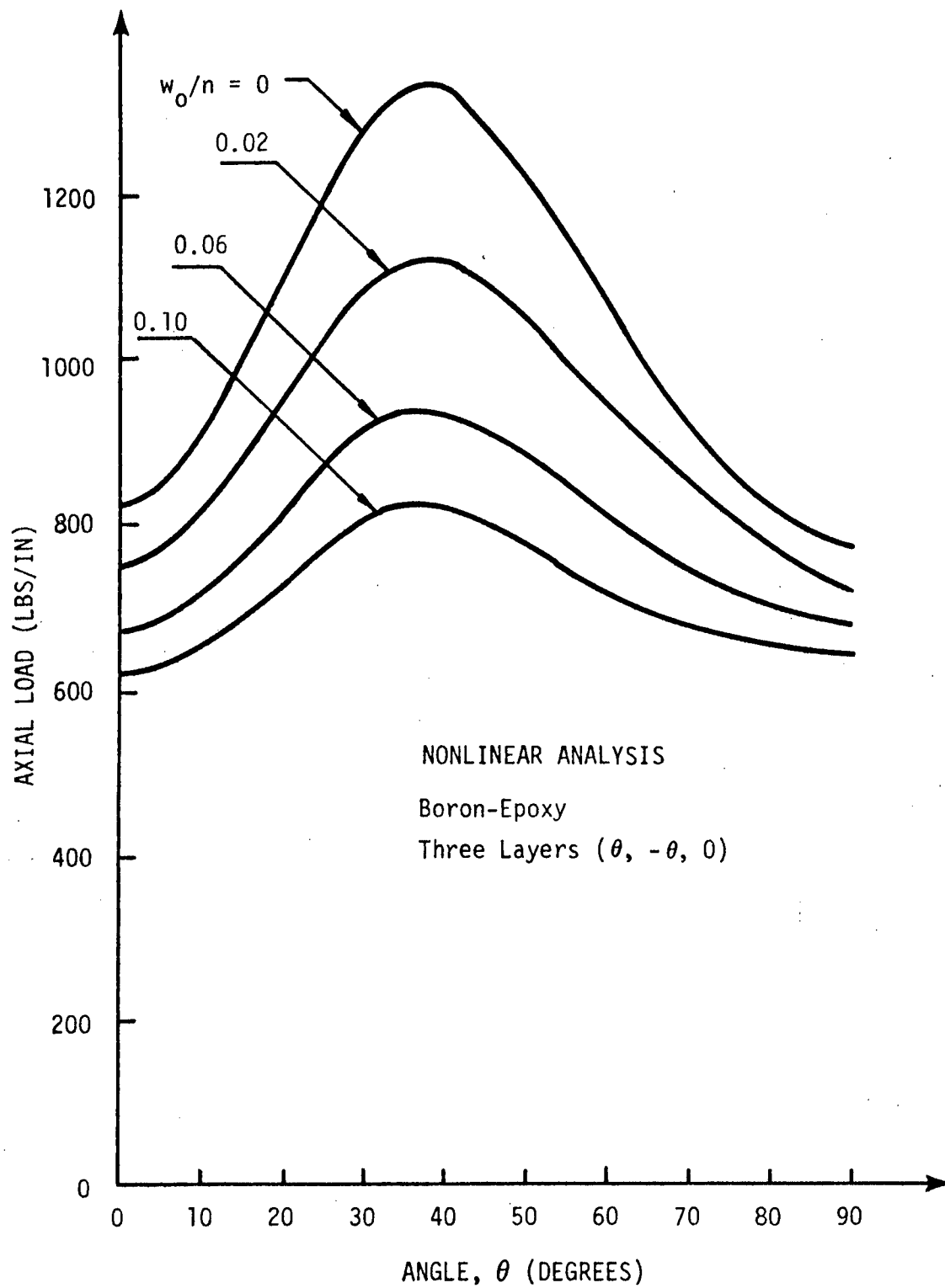
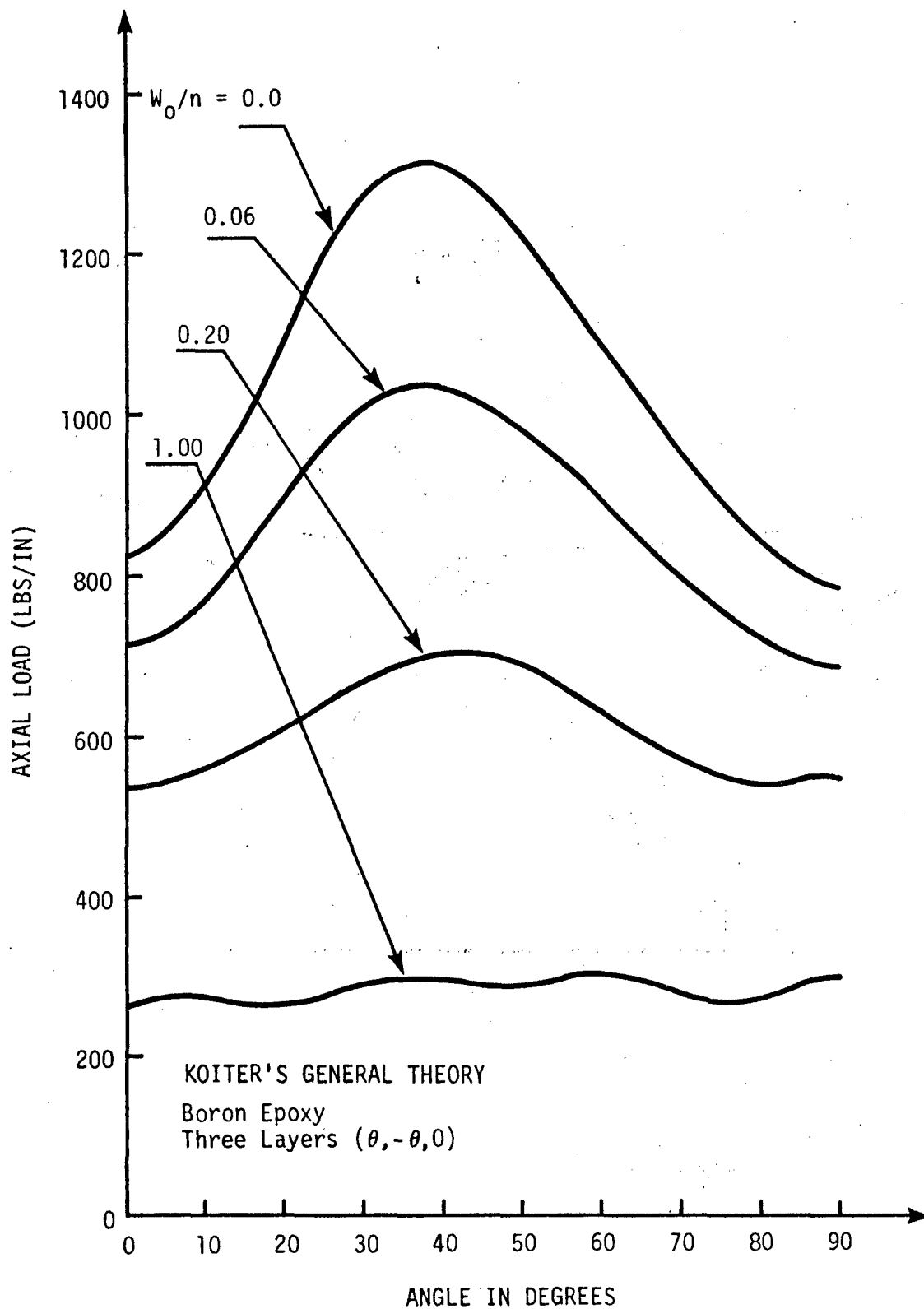


Figure 6 Empirical factor  $\phi$  for cylinders subjected to axial compression.





**Figure 7** Effect of initial imperfection amplitude ( $w_0$ ) on the buckling load of composite cylinder.



**Figure 8** Effect of initial imperfection amplitude  $w_0$  on the buckling load of composite cylinder.

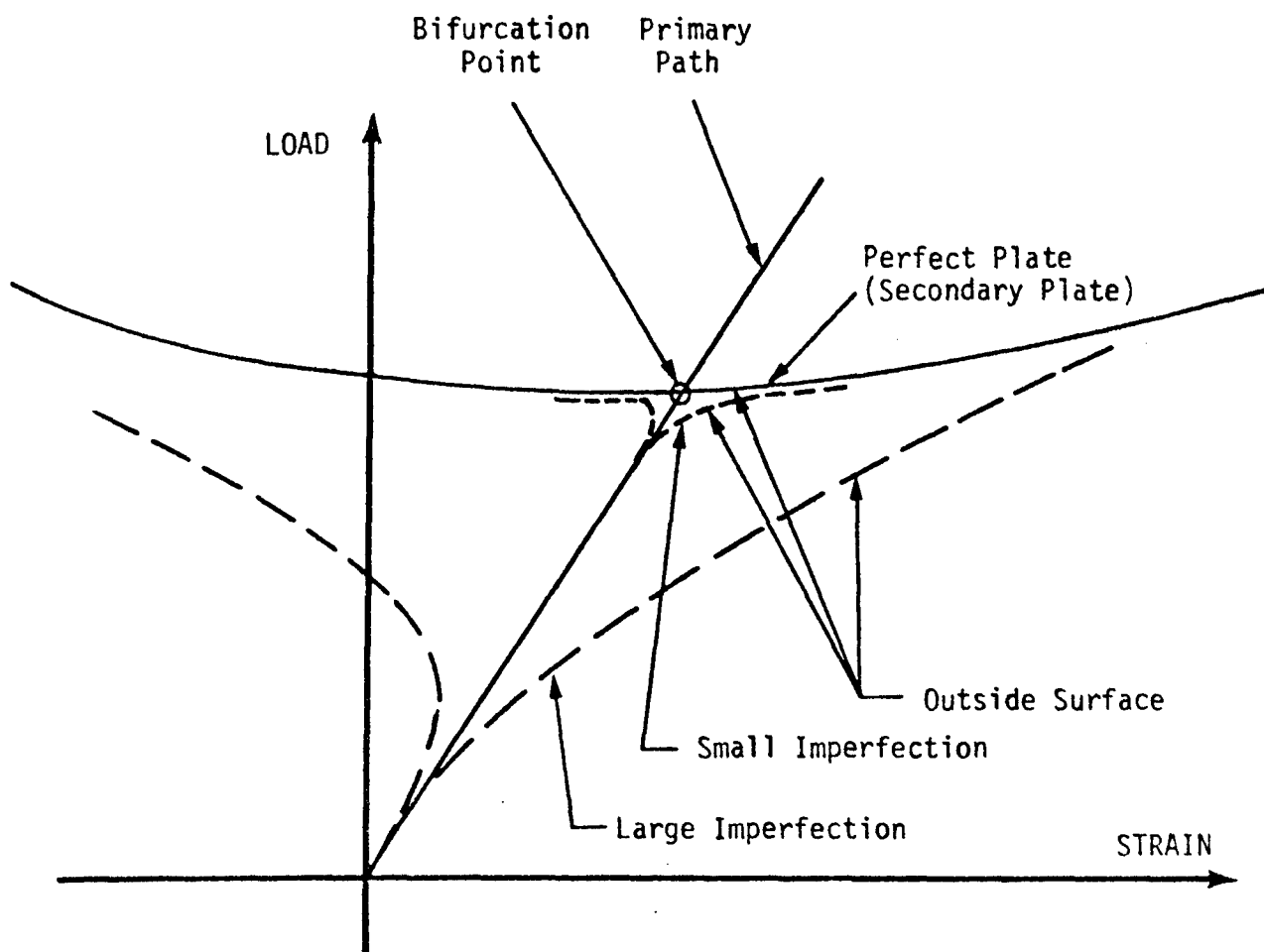


Figure 9 Strain reversal at buckling.

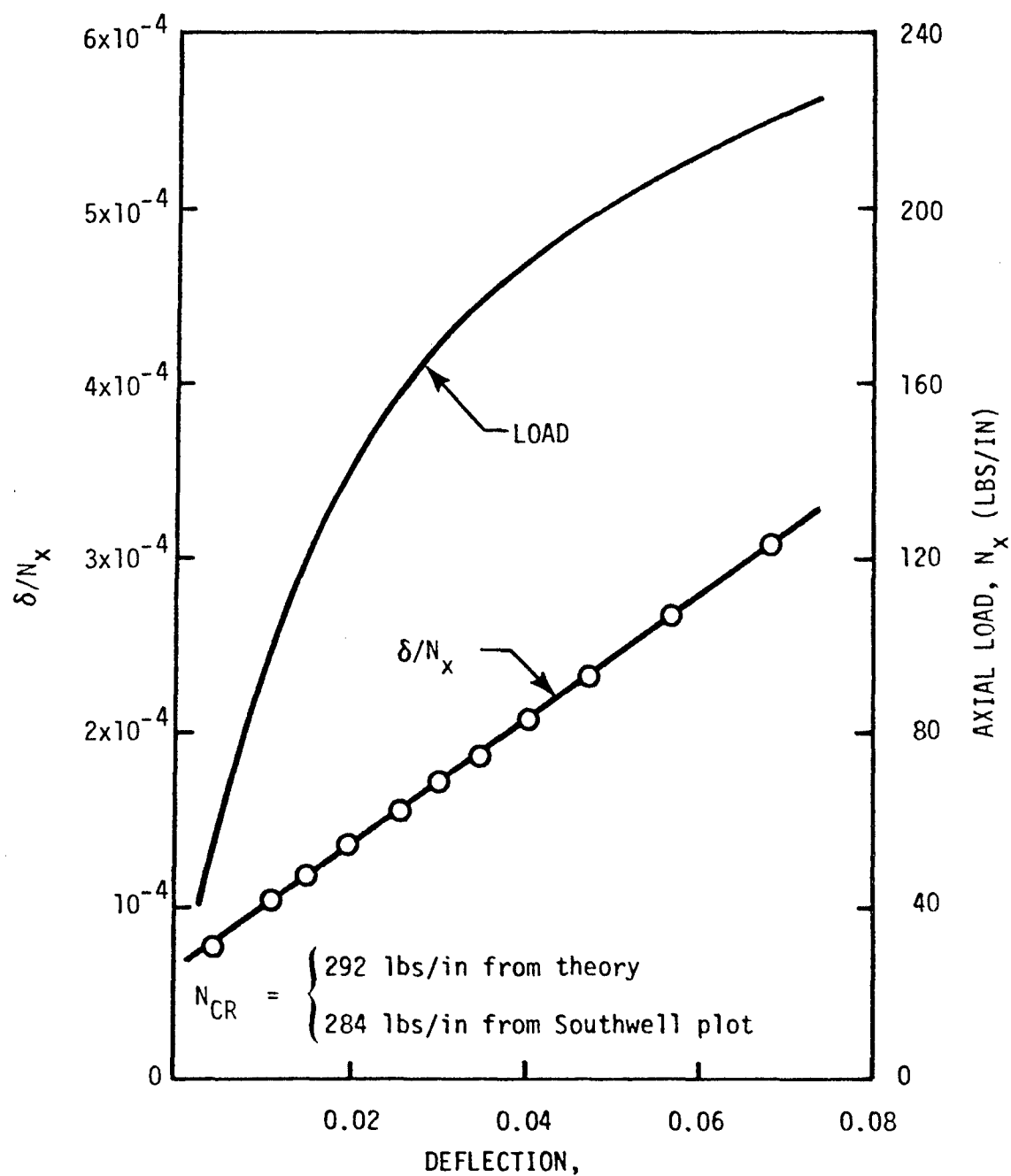


Figure 10 Load-deflection diagram and Southwell plot for simply supported composite plate (from Ref. [18], Test 203b).

## APPENDIX

### BIBLIOGRAPHY ON BUCKLING OF COMPOSITE STRUCTURES

#### A. GENERAL THEORY

The publications in Section A discuss the bifurcation buckling analysis based on a first order shell theory, i.e., the influence of transverse shear deformations is not included.

- A-1      Reissner, E. and Stavsky, Y., Bending and Stretching of Certain Types of Heterogeneous Aelotropic Elastic Plates, J. Appl. Mech., Vol. 28, pp. 402-408, September 1961.

*This paper does not discuss the buckling. However, it appears to represent the first recognition of the importance of the membrane-bending coupling. It is indicated that the plate bending stiffness due to this effect can be considerably reduced.*

- A-2      Dong, S. B., Pister, K. S. and Taylor, R. L., on the Theory of Laminated Anisotropic Shells and Plates, J.A.S., Vol. 29, pp. 969-975, August 1962.

*The paper derives the general theory of buckling of laminated plates. Many of the applications to practical cases presented subsequently were based on these equations.*

- A-3      Cheng, S., and Ho, P. B. C., Stability of Heterogeneous Aelotropic Cylindrical Shells Under Combined Loading, AIAA J., Vol. 1, pp. 892-898, April 1963.

*A completely general set of buckling equations for cylindrical shells is derived. An analytic solution for special boundary conditions is presented.*

- A-4 Tasi, J., Effect of Heterogeneity on the Stability of Composite Cylindrical Shells Under Axial Compression, AIAA J., Vol. 4, pp. 1058-1062, June 1966.

*The theory of Reference A-2 is applied in a study of the buckling of cylindrical shells. The results indicate a deleterious effect of the membrane bending coupling.*

- A-5 Pagano, N. J., and Helpin, J. C., Tension Buckling of Anisotropic Cylinders, J. Composite Matls., Vol. 2, pp. 154-167, April 1968.

*The paper shows by use of theory as well as experiment that if both ends of an anisotropic cylinder are restrained from rotation, an applied axial extension results in the development of inplane shear in the shell wall and eventually in buckling.*

- A-6 Jones, R. M., Buckling of Circular Cylindrical Shells with Multiple Orthotropic Layers and Eccentric Stiffeners, AIAA J., Vol. 6, pp. 2301-2305, December 1968.

*The stability equations, including "smeared stiffeners" are derived and applied to one example. The author warns that the coupling between membrane and bending effects should not be overlooked.*

- A-7 Ashton, J. E., Approximate Solutions for Unsymmetrical Laminated Plates, J. Composite Matls., Vol. 3, pp. 189-191, January 1969.

*The paper presents an approximate method to account for membrane bending coupling by reducing the bending stiffness in an orthotropic analysis.*

- A-8 Whitney, J. M., Shear Buckling of Unsymmetrical Cross-Ply Plates, J. Composite Mat., Vol. 3, pp. 359-363, April 1969.

*A Fourier series solution, including membrane bending coupling, is derived. It is shown that this coupling is important if the plies are few but that the deleterious effect can be practically eliminated in plates with many plies (six or more). For crossplies ( $\theta = 0^\circ, 90^\circ$ ) the reduced stiffness method presented in Reference A-7 gives a good approximation.*

- A-9 Whitney, J. M., and Leissa, A. W., Analysis of Heterogeneous Anisotropic Plates, J. Appl. Mech., Vol. 36, pp. 261-266, June 1969.

*Nonlinear equations including the effect of coupling between membrane and bending action. Subsequently, the equations are linearized and used in a bifurcation buckling analysis of angle-ply plates. The number of plies and the fiber orientation are varied. With only two layers,  $\theta = 45^\circ$  and  $E_1/E_2 = 40$ , the coupling effect reduces the buckling load by a factor of about three.*

- A-10 Chamis, C. C., Buckling of Anisotropic Composite Plates, ASCE, pp. 2119-2139, October 1969.

*A Galerkin solution is presented to the buckling problem for anisotropic plates with simple support and under different types of loading. Effects of membrane bending coupling are included and the reduced stiffness method of Reference A-7 is found to be a good approximation.*

- A-11 Holston, Jr., A., Buckling of Orthotropic Plates With One Free Edge, AIAA J., Vol. 8, 1352-1354, July 1970.

*The theory neglects anisotropy as well as membrane bending coupling.*

- A-12 Kicher, T. P., Predicting Structural Behavior of Composites, 26th Annual Techn. Conf. of the Society of the Plastic Industry, Inc., 1971.

*The paper summarizes some of the work on buckling and vibrations of composite material plates. A warning is issued that the reduced bending stiffness approach (Reference A-7) may not always be accurate.*

- A-13 Jones, R. M., Buckling of Stiffened Multilayer Circular Cylindrical Shells with Different Orthotropic Moduli in Tension and Compression, AIAA J., Vol. 9, pp. 917-922, May 1971.

*The stress-strain curve for the material is assumed to be bilinear with the discontinuity at the origin. An interaction curve for axial load and lateral pressure buckling is computed for a material with the tension modulus twice as large as the compression modulus. Further results on this topic are presented in Reference A-26.*

- A-14 Greszczuk, L. B., and Miller, R. J., Advanced Design Concepts for Buckling Critical Composite Shell Structure, J. Aircraft, Vol. 8, pp. 367-373, May 1971.

*Aluminum cylindrical shells with T-stiffeners are reinforced with strips of boron-epoxy. It is shown that 10% of the composite material by volume can increase the buckling load by as much as 80% for elliptical cylinders under external pressure. A couple of experiments verify the analytical results.*

- A-15 Chamis, C. C., Buckling of Boron/Aluminum and Graphite/Resin Fiber Composite Anisotropic Panels, Natl. Sampe Conf. on Space Shuttle Materials, 1971.

*Theoretical results are presented for the buckling of single-ply rectangular plates. Buckling loads are given as functions of the fiber orientation.*

- A-16 McElman, J. A., Vibration and Buckling Analysis of Composite Plates and Shells, J. Composite Mat., Vol. 5, pp. 529-532, October 1971.



*Toroidal shell segments are analyzed. The affects of membrane bending coupling are demonstrated.*

- A-17 Chamis, C. C., Theoretical Buckling Loads of Boron/Aluminum and Graphite/Resin Fiber Composite Anisotropic Plates, NASA TN D6572, December 1971.

*Essentially the same as Reference A-14. A computer program listing is added.*

- A-18 Hahn, H. T., Nonlinear Behavior of Laminated Composites, J. Composite Mat., Vol. 7, pp. 257-271, April 1973.

*The author observes that the stress-strain curve for shear in single laminates may be highly nonlinear. This may reflect the time dependence of the deformation of the matrix. The results indicate that inclusion of this effect might reduce the buckling load by some 5%. The effect may be of importance when composites are used at elevated temperatures.*

- A-19 Oyler, J. F. and Dym, C. L., The Dynamics and Stability of Composite Shells, Proc. 13th Midwestern Conf., Developments in Mechanics, Vol. 7, pp. 475-488, August 1973.

*The paper considers two different stacking sequences for the wall of cylindrical, barrel shaped, "inversely barrel shaped", and spherical shells. The stacking sequence that gives the highest buckling loads for cylinders and spheres is found to be the least efficient for barrel shaped shells.*

- A-20 Viswanathan, S. B., and Tamekani, M., Elastic Buckling Analysis for Composite Stiffened Panels and Other Structures Subjected to Biaxial Inplane Loads, NASA CR-2216, September 1973.

*The computer program BUCLASP 2 for buckling of panels and including composite material is described. The program applies to cases in which it can be assumed that the buckling mode in the direction of the stiffeners is sinusoidal. Some numerical results are presented. These include some comparisons to analytical solutions. Special problems considered are comparison between bonded and riveted structures and the effects of boron fiber reinforcement of titanium panels.*

- A-21 Viswanathan, A. V., Tamekuni, M., and Tripp, L. L., Elastic Stability of Biaxially Loaded Longitudinally Stiffened Composite Structures, AIAA J., Vol. 11, pp. 1553-1559, November 1973.

*This is an excerpt from Reference A-20.*

- A-22 Stavsky, Y., Thermoelastic Stability of Laminated Orthotropic Circular Plates, Technion, Haifa, Israel, Rept. TDM-73-33, December 1973.

*Circular plates, materially orthotropic with respect to the polar coordinates, are subjected to axisymmetric loading. Buckling loads are presented for a few cases demonstrating some sensitivity to the fiber orientation.*

- A-23 Fortier, R. C., Transverse Vibrations Related to Stability of Curved Anisotropic Plates, AIAA J., Vol. 11, pp. 1782-1783, December 1973.

*This very brief note shows how natural frequencies for 45° angle-ply and cross-ply plates vary as the buckling load is approached.*

- A-24 Jones, R. M., Buckling and Vibration of Unsymmetrically Laminated Cross-ply Rectangular Plates, AIAA J., Vol. 11, pp. 1626-1632, December 1973.

*The effect of membrane bending coupling is illustrated by use of a few examples.*

- A-25      Viswanathan, A. V., Tamekuni, M., and Baker, L. L., Elastic Stability of Laminated, Flat and Curved, Long Rectangular Plates Subjected to Combined Inplane Loads, NASA CR-2330, June 1974.

*Some numerical results from BUCLAP2 on the buckling of long laminated plates are presented. A warning is included that the "reduced bending stiffness method" (Reference A-7) does not always give good results.*

- A-26      Jones, R. M., and Morgan, H. S., Buckling of Stiffened Laminated Composite Circular Cylindrical Shells with Different Moduli in Tension and Compression - Preliminary Results, ONR Contract Rept. NR-064-551, November 1974.

*A buckling criterion is derived for shells with different moduli in tension and compression. The effect of this difference is significant only if the ratio between the moduli is larger, say 2 or so, than it is for most composites. The effect is illustrated on interaction curves where discontinuities occur as the stress in one direction changes sign. Presumably the modulus in any given direction also depends on the stress in the other direction. If such an effect could be included, the interaction curves may be somewhat smoother.*

- A-27      Whitney, J. M., and Sun, C. T., Buckling of Composite Cylindrical Characterization Specimens, J. Composite Mat., Vol. 9, pp. 138-148, April 1975.

*A discussion is presented on the critical buckling loads for typical tubular stress specimen for assessment of strength and stiffness.*

- A-28      Harris, G. Z., The Buckling and Postbuckling Behavior of Composite Plates Under Biaxial Loading, International J. Mech. Sci., Vol. 17, pp. 187-202, 1975.

*The paper elaborates on the applicability of bifurcation theory in view of the lateral displacements caused by membrane bending coupling effects. The bifurcation buckling load is a meaningful parameter only if prebuckling bending is insignificant. It is claimed that the "reduced bending stiffness method" (Reference A-7) is applicable only if the principal directions of the laminates are parallel to the plate edges.*

- A-29 Jones, R. M. and Morgan, H. S., Buckling and Vibration of Cross-Ply Laminated Circular Cylindrical Shells, AIAA J., Vol. 13, pp. 664-671, May 1975.

*The need to include membrane bending coupling effects in buckling and vibration analyses is demonstrated.*

- A-30 Chao, C. C., Koh, S. L., and Sun, C. T., Optimization of Buckling and Yield Strengths of Laminated Composites, AIAA J., Vol. 13, pp. 1131-1132, September 1975.

*The paper considers optimum fiber orientation for angle plies with 20 layers. Plates with different aspect ratios are included.*

- A-31 Hayashi, T., Optimization of the Torsional Rigidity and Strength for Fiber Reinforced Composite Cylinders, Proc. International Conf. on Composite Materials, pp. 703-722, 1975.

- A-32 Housner, J. M., and Stein, M., Numerical Analysis and Parametric Studies of the Buckling of Composite Orthotropic Compression and Shear Panels, NASA TN D-7996, October 1975.

*A large number of numerical results are presented for rectangular orthotropic panels in shear. The variation of shear and compression buckling loads with the fiber orientation is presented for plates of different aspect ratios.*

- A-33 Harris, G. Z., The Buckling of Orthotropic Rectangular Plates, Including the Effect of Lateral Edge Restraint, International J. Solids Struct., Vol. 11, pp. 877-885, 1975.

*A buckling analysis based on a sinusoidal mode is presented. It is pointed out that constraint on transverse displacement can severely reduce the buckling load.*

- A-34 Tennyson, R. C., Buckling of Laminated Composite Cylinders: A Review, J. Composite Mat., Vol. 11, pp. 17-24, January 1975.

*The paper reviews the literature on the problem of buckling of cylindrical shells under different loading conditions. The special problems of nonlinear stress/strain behavior, different moduli in tension and compression and the effects of shape imperfections are discussed. In addition to the review, a series of torsion tests are reported with the experimental results ranging from 0.79 to 1.17 times the analytical. The conclusion is that a moderate reduction factor is sufficient in the case of torsion. In the case of axial compression, a more severe reduction is needed. The authors suggest that many more tests are needed.*

- A-35 Sawyer, J. W., Flutter and Buckling of General Laminated Plates, J. Aircraft, Vol. 14, pp. 387-391, April 1977.

*A Galerkin solution based on beam functions is presented for bifurcation buckling and flutter. The emphasis in the paper is on the effects of anisotropy and fiber-orientation. Both for shear and axial compression, the optimum angle-ply corresponds to a fiber orientation of about  $45^\circ$ .*

- A-36 Simitses, G. J., and Giri, J., Buckling of Rotationally Restrained Orthotropic Plates Under Uniaxial Compression, J. Composite Mat., Vol. 11, pp. 345-364, July 1977.

*A two-term Galerkin solution is used for analysis of critical axial compression of orthotropic plates. Results are given for different degrees of rotational constraint along the edges.*

- A-37      Spier, E. E., Stability Analysis and Testing of Thin-Walled Open-Sectioned Graphite/Thermoplastic Structures, 10th National Sampe Conference, Kiamesha Lake, New York, October 1978.

*Local and general instability failures are considered for a compression member in a lightweight satellite structure of composite material. Results of two crippling tests show good agreement with analysis.*

## B. EFFECTS OF TRANSVERSE SHEAR

The following publications are concerned with the deleterious effects of transverse shear deformation on the buckling load of shells and plates. While the first two are concerned with the theory in general, the remaining papers apply specifically to composite material.

- B-1        Reissner, E., The Effect of Transverse Shear Deformation on the Bending of Elastic Plates, J. Appl. Mech., Vol. 12, pp. A69-A77, June 1945.

*The effect of transverse shear is introduced in the plate bending theory. The shear stress distribution through the thickness is assumed to vary in a way that is consistent with a linear normal stress distribution.*

- B-2        Mindlin, R. D., Influence of Rotatory Inertia and Shear on Flexural Motions of Isotropic Elastic Plates, J. Appl. Mech., Vol. 18, pp. 31-38, March 1951.

*The influence of transverse shear deformation is introduced in a way that is similar to the procedure in Reference B-1.*

- B-3        Spillers, W. R., A Laminated Thin Cylindrical Shell Under Axisymmetric Static Loading, ONR, Techn. Rept. No. 39, April 1967.

*A laminated shell is modeled by representation of bond layers with a set of springs. The springs can deform in the lateral direction and also undergo slip to represent the shear deformation. A few numerical results are given.*

- B-4      Srinivas, S., and Rao, A. K., Buckling of Thin Rectangular Plates, AIAA J., Vol. 7, pp. 1645-1646, August 1969.

*The paper considers the effect on the buckling of isotropic plates of the transverse shear deformation. Results are given for simply supported plates of different aspect ratios.*

- B-5      Durlofsky, H., and Mayers, J., Effects of Interlaminar Shear on the Bending and Buckling of Laminated Beams, USAAVLABS Rept. 70-7, March 1970.

*Shear deformations in the bond layer between different laminates are included in an analysis of bending and buckling of laminated beams.*

- B-6      Srinivas, S., Joga Rao, C. V., and Rao, A. K., Some Results from an Exact Analysis of Thick Laminates in Vibration and Buckling, J. Appl. Mech., Vol. 37, pp. 868-870, September 1970.

*Equations for vibration and buckling are derived for plates consisting of isotropic lamina. Some results are given for three-ply laminates. If the shear modulus is the same in all lamina and  $h/b = 0.1$ , the error due to neglect of transverse shear is about 7%. If the shear stiffness in the middle layer is reduced by a factor of 15, the error is 35%.*

- B-7      Srinivas, S., and Rao, A. F., Bending Vibration and Buckling of Simply Supported Thick Orthotropic Rectangular Plates and Laminates, International J. Solids, Struct., Vol. 6, pp. 1463-1481, 1970.

*The theory of Reference B-6 is extended to orthotropic laminates. It is shown by comparison to 3-dimensional analysis that the second order shell theories by Reissner and Mindlin (References B-1 and B-2 give good results for very thick plates.*



- B-8 Brunelle, E. M., Elastic Instability of Transversely Isotropic Timoshenko Beams, AIAA J., Vol. 8, pp. 2271-2273, December 1970.

*The effect of transverse shear on the buckling of composite material columns is studied. Results are shown in the range of  $20 \leq E_1/G \leq 50$ .*

- B-9 Durlofsky, H., and Mayers, J., The Effects of Interlaminar Shear on the Bending and Buckling of Fiber-reinforced Composite Flat and Curved Panels, USAAVLABS Rept. 71-10, May 1971.

*A theory is derived for buckling of laminated shells in which the transverse shear deformation is included and the corresponding modulus can vary from layer to layer. Buckling loads are presented as functions of  $G/E_1$  in the range of 0.01 - 1.0.*

- B-10 Brunelle, E. J., Buckling of Transversely Isotropic Mindlin Plates, AIAA J., Vol. 9, pp. 1018-1022, June 1971.

*Mindlin's second order theory (Ref. B-2) is used in analysis of plates of width  $b$  loaded in the axial direction. The aspect ratio  $a/b$  and the parameter  $S \sim (E_1/G_T) (h/b)^2$  are varied in a parametric study.*

- B-11 Noor, A. K., Stability of Multilayered Composite Plates, Fiber Science and Technology, Vol. 8, pp. 81-89, 1975.

*A second order theory (Reissner-Mindlin) is derived. Results from this theory are compared to results from three-dimensional theory. It is found that the plate theory can give good approximations for plates with  $b/h = 0.3$ . It is also found that the correction for shear deformation varies with the number of layers. With moduli typical for glass/epoxy composites ( $E_1/E_2 = 30$  and the transverse shear moduli 0.5 and 0.6 times  $E_2$ ) it is found that the effect should be included if  $h/b > 0.05$ .*

- B-12      Noor, A. K., and Mathers, M. D., Anisotropy and Shear Deformation in Laminated Composite Plates, AIAA J., Vol. 14, pp. 282-285, February 1976.

*For the purpose of examining the effects of transverse shear, it adds little to the results of Reference B-11.*

- B-13      Noor, A. K., Mathers, M. D., and Anderson, M. S., Exploiting Symmetries for Efficient Postbuckling Analysis of Composite Plates, AIAA J., Vol. 15, pp. 24-32, January 1977.

*The paper is primarily concerned with computer economy. In regard to the effects of transverse shear deformation, it is noted that this effect in the postbuckling range is similar to its effect on the buckling load. That is, it should be included for a typical (simply supported plate) if  $h/b > 0.05$ .*

### C. IMPERFECTION SENSITIVITY AND POSTBUCKLING STRENGTH

The following publications discuss the imperfection sensitivity of the buckling load and possible postbuckling strength for plates and shells in compression. Except for the first three papers on the general theory, they consider the special problems connected with use of composite materials.

- C-1 Koiter, W. T., On the Stability of Elastic Equilibrium (in Dutch), thesis, Delft, H. J. Paris Amsterdam, 1945. English translation, AFFDL Rept. TR-70-25, 1970.
- C-2 Brush, D. O., and Almroth, B. O., Buckling of Bars, Plates, and Shells, McGraw-Hill, 1975.
- C-3 Koiter, W. T., The Effect of Axisymmetric Imperfections on the Buckling of Cylindrical Shells Under Axial Compression, Proc. K. Ned. Akad. Wet., Amsterdam, Ser. B, Vol. 6, 1963; also, Lockheed Missiles and Space Company, Rept. 6-90-63-86, August 1963.
- C-4 Khot, N. S., On the Effects of Fiber Orientation and Nonhomogeneity on Buckling and Postbuckling Equilibrium Behavior of Fiber-reinforced Cylindrical Shells Under Axial Compression, AFFDL, TR-68-19, May 1968.

*An analysis is presented of buckling and postbuckling behavior of simply supported cylindrical shells of composite material. A Donnell type theory including the effects of anisotropy is used. The deformation pattern in the postbuckling range is represented by a four term series. Numerical results are obtained for a number of different three-layered shells. The maximum efficiency is obtained with the combinations  $\theta = (0^\circ, 70^\circ, -70^\circ)$  and  $(90^\circ, 20^\circ, -20^\circ)$ .*

- C-5 Chehill, D. S., and Cheng, S., Elastic Buckling of Composite Cylindrical Shells Under Torsion, J. of Spacecraft, Vol. 5, pp. 973-978, August 1968.

*A nonlinear analysis is formulated for anisotropic shells with imperfections under torsional loading. The numerical results indicate moderate imperfection sensitivity.*

- C-6 Khot, N. S., On the Influence of Initial Geometric Imperfections on the Buckling and Postbuckling Behavior of Fiber-Reinforced Cylindrical Shells Under Uniform Axial Compression, AFFDL-TR-68-136, October 1968.

*The same deformation mode as in Reference C-3 is here applied in a nonlinear analysis of axially compressed cylinders with geometric imperfections. Shells with close to optimum fiber orientation appear to be the more sensitive, but within the range under investigation the optimum orientation remains unchanged.*

- C-7 Khot, N. S., Postbuckling Behavior of Geometrically Imperfect Composite Cylindrical Shells Under Axial Compression, AIAA 7th Aeros. Sci. Meeting, New York City, New York, January 1969.

*The paper contains a summary of the results presented in Reference C-3.*

- C-8 Khot, N. S., Venkayya, V. B., and Berke, L., Buckling and Postbuckling Behavior of Initially Imperfect Orthotropic Cylindrical Shells Under Axial Compression and Internal Pressure, IUTAM Conf. on Instability of Continuous Systems, September 1969.

*The analysis of Reference C-3 is extended through the addition of an internal pressure. It is shown that the internal pressure has a beneficial effect on the imperfection sensitivity. For the cases investigated, the Donnell and Sanders equations give essentially the same results.*

- C-9 Khot, N. S., Buckling and Postbuckling Behavior of Composite Cylindrical Shells Under Axial Compression, AIAA J., Vol. 8, pp. 229-235, February 1970.

*The paper presents some of the results in Reference C-6 in a more readily available source.*

- C-10 Khot, N. S., Postbuckling Behavior of Geometrically Imperfect Composite Cylindrical Shells Under Axial Compression, AIAA J., Vol. 8, pp. 579-581, March 1970.

*It is concluded in the paper that composite shells may be somewhat less sensitive to imperfections than the isotropic shells are. Consistently with this observation it is found that boron-epoxy shells are less sensitive than glass-epoxy shells.*

- C-11 Jones, R. M., Buckling of Stiffened Two-Layered Shells of Revolution with a Circumferentially Cracked Unbonded Layer, AIAA J., Vol. 7, pp. 1511-1516, 1969.

- C-12 Almroth, B. O., Burns, A. B., and Pittner, E. V., Design Criteria for Axially Loaded Cylindrical Shells, J. of Spacecraft, Vol. 7, pp. 714-720, June 1970.

*A method is presented to determine the "knockdown factor" for different types of cylinders based on their relative sensitivity to axisymmetric imperfections (Koiter's special theory). For composite material cylinders (filament wound), it appears that the method gives results that are only slightly less conservative than those obtained through application of the same knockdown factor as for an isotropic cylinder with the same "effective thickness."*

- C-13 Khot, N. S., and Venkayya, V. B., Effect of Fiber Orientation on Initial Postbuckling Behavior and Imperfection Sensitivity of Composite Cylindrical Shells, AFFDL Rept. TR-70-125.

*In this paper the authors base their analysis on Koiter's general theory. In comparison to Reference C-6, shells with larger imperfection amplitude are included in the study. With an imperfection of about the size of the shell thickness, the advantage of using a fiber orientation close to optimum is totally eliminated.*

- C-14 Tennyson, R. C., Chan, K. H., and Muggeridge, D. B., Effect of Axisymmetric Shape Imperfections on the Buckling of Laminated Anisotropic Circular Cylinders, Transactions Canadian Aeronautics & Space Institute, Univ. of Toronto, Canada, Vol. 4, pp. 131-139, September 1971.

*The effect of axisymmetric imperfections is studied by use of Koiter's special theory. The numerical study is restricted to the three-layered combinations  $(\theta, 0^0, -\theta)$  and  $(\theta, -\theta, 0^0)$ , of which the latter appear to be the more efficient. As in Reference C-9, it is shown that shells with optimum fiber orientation are the most sensitive to imperfection and that a moderate imperfection amplitude is sufficient to wash out the advantages of an optimum design.*

- C-15 Prabhakara, M. K., and Chia, C. Y., Postbuckling Behavior of Rectangular Orthotropic Plates, J. Mech. Eng. Sci., Vol. 15, pp. 25-33, January 1973.

*A buckling and postbuckling analysis based on a double Fourier-series approach is presented for flat plates. Some plots are given that indicate outer surface stresses as functions of applied load.*

- C-16 Tennyson, R. C., and Muggeridge, D. B., Buckling of Laminated Anisotropic Imperfect Circular Cylinders Under Axial Compression, J. of Spacecraft, Vol. 10, pp. 143-148, February 1973.

*A number of cylinders were measured to determine initial deviations from true geometry. The axisymmetric part of the imperfections was isolated and Koiter's special theory applied. The same results are also presented in Reference D-2 with some additional detail.*

- C-17 Kulkarni, S. V., and Frederick, D., Buckling of Partially Debonded Layered Cylindrical Shells, AIAA/ASME/SAE 14th SDM Conf., Williamsburg, Va., March 1973.

*A local buckling analysis is applied to the two separate branches corresponding to an axisymmetric delamination at midplane.*

- C-18 Chia, C. Y., and Prabhakara, M. K., Postbuckling Behavior of Unsymmetrically Layered Anisotropic Rectangular Plates, J. Appl. Mech., Vol. 41, pp. 155-162, March 1974.

*This represents an extension of the investigation reported in Reference C-15.*

- C-19 Banks, W. M., The Post-Buckling Behavior of Composite Panels, Proc. International Conf. on Composite Matl., Vol. 2, Switzerland, 1975.

*In contrast to References C-15 and C-18, this paper considers the case in which the applied shortening is uniform rather than the force intensity. The postbuckling analysis is based on the assumption that the lateral displacement varies sinusoidally in the axial direction (the direction of the load) and can be accurately defined by a 4-term power series in the transverse direction. A couple of test results were included showing good agreement between computed and measured lateral displacements. The agreement on the stress distribution deteriorates with increasing load.*

- C-20 Chandra, R. Postbuckling Analysis of Crossply Laminated Plates, AIAA J., Vol. 13, pp. 1388-1389, October 1975.

*A one term solution is presented for the postbuckling behavior of unsymmetrically stacked cross-plyies.*

- C-21      Stroud, W. J., Anderson, M. S., and Hennessy, K. W., Effect of Bow-Type Initial Imperfections on the Buckling Load and Mass of Graphite-Epoxy Blade-Stiffened Panels, NASA TM-74063, August 1977.

*Blade-stiffened composite material panels under axial compression are designed for minimum weight with buckling constraints. The bending stresses due to a bow-type imperfection are included. It is found that the weight efficiency is much better if the imperfection is allowed to affect the optimized design.*

- C-22      Banks, W. M., Harvey, J. M., and Rhodes, J., The Nonlinear Behavior of Composite Panels with Alternative Membrane Boundary Conditions on the Unloaded Edges, Dept. of Mech. of Mat., U. of Strathclyde Rept., January 1978.

*In comparison to the results of Reference C-19, an additional term is added to describe the displacement as a function of the axial coordinates. There is no indication of whether this improves correlation with test.*



#### D. EXPERIMENTAL VERIFICATION

The papers in the following are mainly devoted to the verification of analytical results by use of experiments.

- D-1 Ravenhall, R., Stiffness and Buckling in Filament-Wound Motors, J. Spacecraft, Vol. 1, pp. 260-263, August 1964.

*Filament winding is not representative for modern manufacturing techniques of composite shells. The comparison between test and theory may not be quite the same for modern specimens. From the theoretical part of the paper is concluded that the best efficiency is obtained if the balance between helical and hoop windings is such that  $0.5 \leq E_X/E_Y \leq 1.0$ . Four cylinders were tested and the reduction factor,  $\phi$ , was found to be much the same as for metal cylinders. The results were:*

$R/t$	$E_X/E_Y$	$\phi$
152	0.62	0.655
128	0.83	0.630
123	1.26	0.685
122	1.93	0.688

- D-2 Tasi, J., Feldman, A., and Stang, D. A., The Buckling Strength of Filament-Wound Cylinders Under Axial Compression, NASA CR-266, July 1965.

*The cylinders were manufactured through winding. The inplane stiffnesses were determined experimentally through application of lateral pressure, torsion, and tension. The shells were essentially orthotropic. Presumably the number of layers was large enough so the shell wall material could be considered to be homogeneous and orthotropic. Four of the*

cylinders buckled between 40 and 48 percent of the classical buckling load. This severe reduction was assumed to have been caused by pretest damage at the ends. For the other ten cylinders, the knockdown factor varied from 0.66 ( $R/t = 238$ ) to 0.85 ( $R/t = 85$ ). These results are in line with the behavior of isotropic shells.

D-3 Holston, Jr., A., Feldman, A., and Stang, D. A., Stability of Filament-Wound Cylinders Under Combined Loading, AFFDL Rept. TR-67-55, May 1967.

The cylinders were manufactured through winding. All cylinders had a thickness of 0.035 in. with the three different layups.

$0^{\circ}, 45^{\circ}, -45^{\circ}, 90^{\circ}$	labeled isotropic
$0^{\circ}, 22.5^{\circ}, -22.5^{\circ}, 90^{\circ}$	labeled orthotropic
$30^{\circ}, 90^{\circ}$	labeled anisotropic

For each of the three layups and three different radii (3, 6, and 12 in.) five cylinders were manufactured so that for all different geometrical configurations test could be performed in

- axial compression
- torsion
- bending
- combination of axial load and torsion
- combination of axial load and bending

The results of torsion tests can only be explained by the assumption that the theory used underestimates the critical torque approximately by a factor of two. Possibly this is due to inaccuracy in applied boundary condition. All the configurations give approximately the same theoretical buckling load. There is not clear trend indicating a variation in the knockdown factor, not even with the radius/thickness ratio. Typical results are

	Axial Compression		
	Knockdown Factor with $R=$		
	6	12	24
Isotropic	.677	.782	.702
Orthotropic	.899	.773	.719
Anisotropic	.791	.765	.679

*With a larger number of specimens it seems that at least a more pronounced tendency would be found towards decreasing knockdown factor with increasing radius.*

- D-4      Mandell, J. F., An Experimental Investigation of Anisotropic Fiber Reinforced Plastic Plates, AFML TR-68-281.

*Experimental buckling loads are determined for a large number of plates, of which the majority were of composite material with graphite, glass or boron fiber reinforcement. In almost all cases the load displacement curve indicates large lateral displacements and the test is interrupted before the theoretical buckling load is approached. This behavior is observed also for metal plates and composites with little or no membrane bending coupling. Therefore, the eccentricity seems to be due to initial curvature or offset in loading rather than to membrane bending coupling.*

- D-5      Leone, E. M., Oplinger, D. W., and Serpico, J. C., Experimental Studies of the Elastic Stability of Three-Dimensionally Reinforced Composite Shells, AIAA 7th Aerospace Sciences Meeting, New York, January 1969.

*A total of seven cylinders were tested. The material featured graphite fibers in axial and radial directions and quartz filaments for radial reinforcements. The radius-thickness ratio was in the range of 35 to 40. The loading was lateral pressure, hydrostatic pressure, or hydrostatic pressure with the addition of a component of axial compression. With use of experimentally determined inplane shear modulus, predicted lateral pressures agree well with experimental results. The tests including an axial load component seem to fall somewhat below prediction although conservatively a linear interaction curve was used and the selected example indicates that strain reversal may have occurred well below final collapse.*

- D-6 Ashton, J. E., and Love, T. S., Shear Stability of Laminated Anisotropic Plates.

*Shear buckling tests were performed on 14 boron epoxy laminated plates. The load was applied as a diagonal tension on a "rigid picture frame." The corners on the panel were notched so that local failure would be avoided. The critical load is determined by use of the Southwell plot. The load displacement curve is given for one example and indicates a gradual growth of the lateral displacement. As the specimens were symmetric with 16 layers, this is not the result of membrane-bending coupling but rather it must be assumed that the initial geometric deviations were relatively large. As must be expected, the agreement between the Southwell load and analytical (Rayleigh-Ritz) results is good. The failure load (35,800 lb) is given only for one panel (with a buckling load of 29,000 lb). For all panels b/t was about 100.*

- D-7 Kicher, T. P. and Mandell, J. F., An Experimental Study of the Buckling of Anisotropic Plates, AIAA/ASME 10th SDM Conference, April 1969.

Kicher, T. P., and Mandell, J. F., A Study of the Buckling of Laminated Composite Plates, AIAA J., Vol. 9, p. 605, April 1971.

*Both these publications summarize the results of Reference D-4. The article in the AIAA Journal is somewhat more specific about test setup.*

- D-8 Wall, Jr., L. D., and Card, M. F., Torsional Shear Strength of Filament-Wound Glass-Epoxy Tubes, NASA TN D-6140, August 1971.

*The tests on torsion tubes were intended as fracture tests. However, it is observed in many cases that fracture was preceded by strain reversal. Evaluation of the results seems difficult because the stress-strain curves show considerable nonlinearity at low stress.*

- D-9 Willey, B. T., Instability of Glass-fiber Reinforced Plastic Panels Under Axial Compression, ASAAVLABS, Techn. Rept. 69-48, September 1971.

Some 70 flat plates, approximately square with a side of 6-1/4 in., were tested under axial compression. The plates were manufactured from cloth and very thin,  $b/t = 175$  or so. The report gives the Southwell load for all panels. Excessive scatter probably indicates nonuniform material properties. It is stated that the agreement between the Southwell load and observed strain reversal is poor. The author's conclusion that the strain-reversal technique has no value is dubious. Strain reversal is a definite indication of the development of buckles and as such a physically meaningful parameter. On the other hand, any experimental method (whether in agreement with theory or not) that gives a different result is not applicable to the case in question. It may be noticed that at least some of the panels were tested up to 3 or 4 times the bifurcation buckling load.

- D-10 Kaminski, B. E., and Ashton, J. E., Diagonal Tension Behavior of Boron Epoxy Shear Panels, J. Comp. Mat., Vol. 5, pp. 553-558, October 1971.

An experimental analysis is presented of the postbuckling behavior of shear panels. The panel dimensions were 6x18 in. and the thickness about 0.02 in., i.e.,  $b/t = 300$ . The panels were tested to failure which occurred at 8 to 10 times the bifurcation buckling load. In a couple of cases, the panels were tested without failure 100 times to a load level corresponding to 5 times the bifurcation load. Subsequently, these panels were tested to failure with no obvious reduction in strength. Generally, failure occurred in one of the corners of the panel before the average shear strain in the panel had reached half of the shear strain corresponding to fracture.

- D-11 Tennyson, R. C., Muggeridge, D. B., and Chen, K. H., Buckling of Fiber-reinforced Circular Cylinders Under Axial Compression, AFFDL TR-72-102, August 1972.

A number of glass-epoxy cylinders were manufactured with considerable care. The initial geometrical imperfections were measured. The axisymmetric part of these imperfections was isolated and expressed in

terms of trigonometric series. By use of the classical bifurcation buckling analysis, critical loads were computed and the cylinders subsequently were tested. The agreement between computed and measured buckling loads is fair. For the 14 cylinders tested, predictions range from 0.50 to 0.85 times the critical load for perfect cylinders while the experimental results range from 0.41 to 0.75 times this load. In general, the predictions were 10 to 20% on the high side.

- D-12 Carri, R. L., Buckling Behavior of Composite Cylinders Subjected to Compressive Loading, NASA CR-132264, June 1973.

Four boron-epoxy and four titanium cylinders with boron-epoxy reinforcement were tested in axial compression, all with  $R/t$  about 25. Inside and outside strains are recorded as functions of the load for the four composite material cylinders and for one of the titanium cylinders. Only for the titanium cylinder is there some indication of buckling (strain reversal) before failure occurs.

- D-13 Wilkins, D. J., Compression Buckling Tests of Laminated Graphite-Epoxy Curved Panels, AIAA, 12th Aerospace Sci. Meeting, Washington, D.C., January 30-February 1, 1974.

A number of cylindrical panels were tested under axial compression. Some of the test specimens suffered no damage and could be retested under different conditions. Therefore, 72 panels yielded 84 test results. All panels had a length of 13 in., a radius of 12 in., and a width of 9 in. (43°). Panel thickness and layup were varied through the test series.

Buckling loads were registered by use of the Moire grid technique. For flat panels it appears that initial eccentricities or membrane bending coupling would render this technique useless; compare the results of Reference D-4. However, for the cylindrical panels, it appears that the development of the buckling pattern is rapidly accelerated as the critical load is reached and the Moire technique

seems to have been very useful. In some cases, the tests were continued after the Moire pattern indicated buckling. In no case could the load be significantly increased before snap-through occurred.

Typical examples are:

<u>Moire</u>	<u>Snap-through</u>
6300	6340
715	725
1290	1315
5700	5810
3530	3685

In nine cases snap-through occurred without previous indication by the Moire pattern. In five cases the indication by the Moire pattern was essentially simultaneous with snap-through. In the remaining cases the test was interrupted when the Moire pattern indicated buckling and the specimens could be used again.

Curve A in Figure 1 gives on the vertical axis the percentage of the specimen for which the critical load exceeds the corresponding load (as fraction of the critical load for a perfect cylinder) on the horizontal axis. It may be noted that the reduction factor is of the same order as it would be for isotropic cylinders.

In addition to the observation of snap-through loads and Moire patterns, Southwell plots are presented for 59 of the cylinders. The information from those serves primarily for evaluation of this method as a procedure for nondestructive testing. Due to the imperfection sensitivity, the initial buckling is likely to be local in character. Therefore, the Southwell plot is meaningful only if the position of the reference displacement is appropriately selected. In the present report, the lateral deformation is scanned in a preliminary test to a relatively high load level. The point with the greatest growth at that load is selected as a reference point. In most cases, this seems to work well, but there are cases in which it fails as indicated by the following results:

Southwell	Theory	Test (Moire)
7323	7200	6680
16625	16200	14680
14118	12500	12460

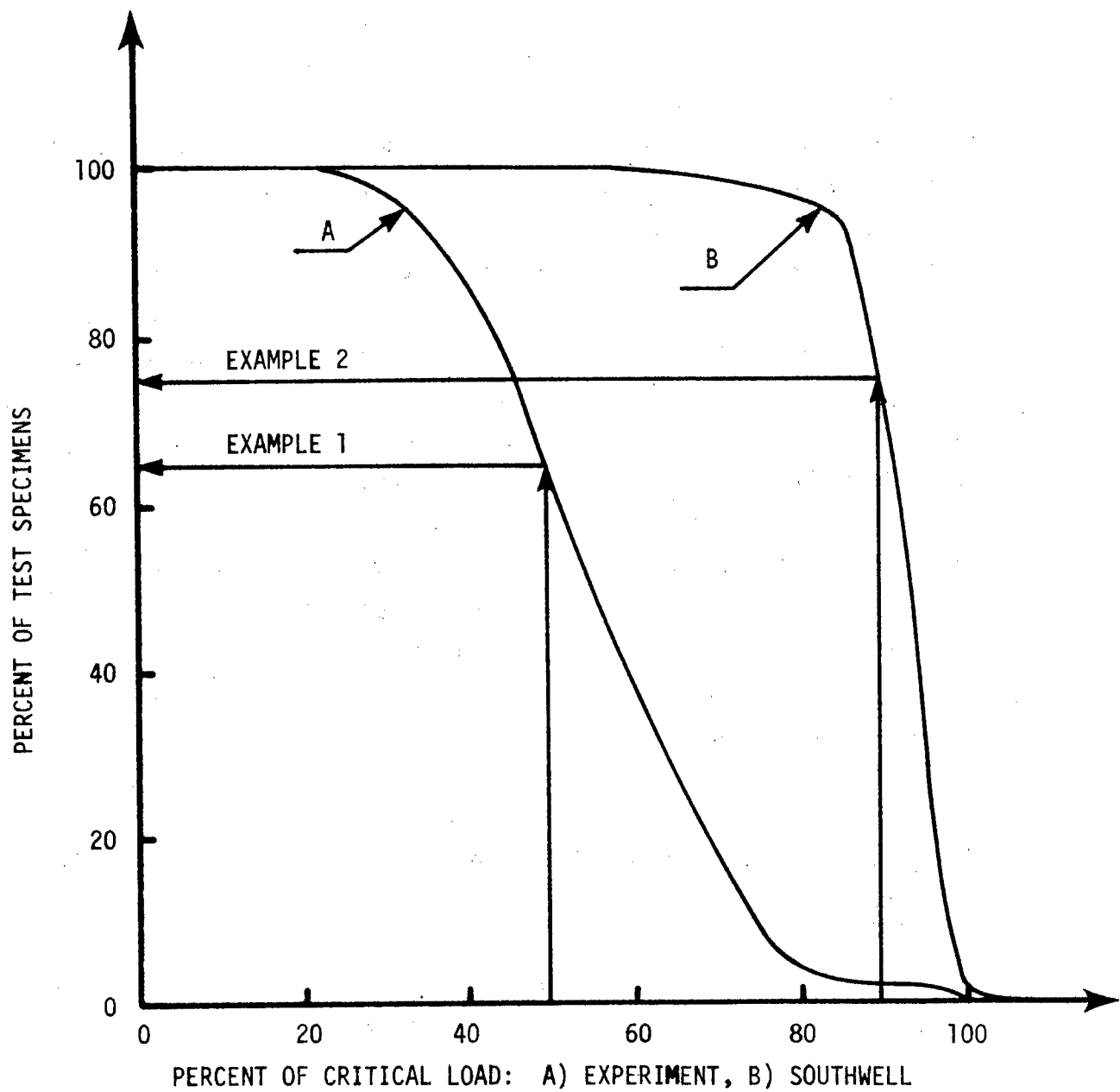
When the Southwell plot fails to give an accurate prediction of the critical load, it is unconservative because the most severe imperfection has little effect on the deformation of the reference point. Curve B in the figure shows how many percent of the specimens carried a load exceeding a given fraction of the experimental buckling load based on the Southwell approach. The method is sometimes inaccurate, for example, the critical load is overestimated by 10% or more in 25% of the cases and by 20% or more in 4%. On the other hand, very few failures would occur in "nondestructible testing" if reasonable caution is exercised. The Moire method determines the critical load with accuracy, but cannot be relied upon for nondestructive testing. The present results indicate that no more than about 80% of the specimens could be saved. For more complex structures, the selection of a reference point for the load displacement diagram makes nondestructive testing difficult in any case.

- D-14 Wilkins, D. J. and Love, T. S., Combined Compression-Torsion Buckling Tests of Laminated Composite Cylindrical Shells, AIAA/ASME/SAE 15th SDM Meeting, Las Vegas, April 1974.

Six cylinders were manufactured for the tests. The laminates were four-ply  $(\pm 45)_S$  and six-ply  $(0/\pm 45)_S$ . The outside diameter was 15 in. and the cylinder length 15 in. The ply thickness varied from 0.0053 to 0.0065. Strain reversal was used to indicate buckling and in most cases the test was nondestructive so that a complete or partial interaction curve could be obtained for one specimen. The knockdown factor for axial compression is about 0.65. The analytical solution for torsion buckling appears to be off approximately by a factor of two.

- D-15 Wilkins, D. J., and Olson, F., Shear Buckling of Advanced Composite Curved Panels, SESA Fall Meeting, Seattle, Wash., October 17-20, 1972.





EXAMPLE 1: 65% of all test observations based on the Moiré method failed above 50% of the classical buckling load.

EXAMPLE 2: 75% of all test observations fell at or above 90% of the estimate by the Southwell method.

Figure 1 Test results from Reference D-13.

Four graphite-epoxy and four boron-epoxy cylindrical panels were tested in shear. The panels covered each  $45^\circ$  and four panels were mounted together. The shell wall consisted of 8 ply  $\pm 45^\circ$  laminate. Due to the fact that relatively few laminates were used, the critical torque is about 50% higher in the favorable direction. The panels were 9 in. long and the radius was 12 in. The average thickness was about 0.056 (R/t about 200). The specimens were first tested in the weaker direction. The buckling load was registered by use of a Southwell plot based on bending strain rather than displacement. The critical load so determined varied between 84 and 106 percent of the theoretical load corresponding to clamped edges. In a second test, the torque was applied in the opposite direction. The Southwell load varied between 85% and 95% of the theoretical value for clamped shells. Furthermore, the Southwell loads agreed very well with the load level at which the lateral displacements started to grow rapidly. The tests in this direction were continued to failure which occurred at about 20% above the buckling load. At this load level, the buckle depth was of the same order of size as the shell thickness.

D-16 Marlowe, D. E. and Sushinsky, G. F., Torsional Buckling of Composite Cylindrical Shells, National Bureau of Standards, Rept. NBSIR 74-572, September 1974.

Of 39 cylinders tested, a few were pure metal cylinders, a few composite reinforced metal cylinders, but the majority were pure composite cylinders. Since many of the cylinders were tested in both directions, a total of 51 test results were available for comparison to theory. Boron-epoxy and graphite-epoxy cylinders were tested with various layups and for two different R/t values (150 and 75) and three different values of L/R (3.2, 6.7, 13.3). Due to the great variety of specimens, duplication is minimal which makes it difficult to discover any clear trends. There is, for example, no indication that the parameters L/R or R/t should have any influence on the knockdown factor  $\phi$ . It may be noticed, however, that two tests with R/t = 75 and with a

favorable layup fractured without any previous indication of buckling at about 60% of the critical load. An optimum layup was determined analytically for shells subjected to torsion in one direction only. For these cylinders  $(-82.5, 30, 20, -82.5)_S$ , the knockdown factors were relatively low (0.79, 0.88, 0.61, 0.54). For this layup the ratio between critical loads for loading in the two directions is as high as 3.5. For shells with  $45^\circ$  windings only this ratio is of the order 1.5 to 2.0. There is no indication among these that the more efficient layups should be more sensitive to imperfections. Generally the torque drops somewhat after buckling or it stays constant. In no case is there any indication of postbuckling strength above the bifurcation load. The strain at fracture is not observed but stress-strain curves are shown for all specimens. In many of these (including some with  $R/t = 75$ ), strain levels were reached which considerably (5 times or so) exceed the critical strain. The knockdown factor for the 51 experiments varies from 0.54 to 0.95 with an average close to 0.80.

- D-17 Marlowe, D. W., Sushinsky, G. F., and Dexter, H. B., Elastic Torsional Buckling of Thin-Walled Composite Cylinders, Composite Materials, Testing and Design (Third Conf.), ASTM STP 546, ASTM 1974.

The results of Reference D-16 are presented in a slightly more polished form.

- D-18 Chailleux, A., Hans, Y., and Verchery, G., Experimental Study of the Buckling of Laminated Composite Columns and Plates, Int. J. Mech. Sci., Vol. 17, p. 485, 1975.

Composite flat plates were tested under different compression. The value of  $b/t$  was about 100 for all plates. Most plates have considerable membrane-bending coupling. Load displacement curves are not shown but it can be assumed that the bending of the plate sets in gradually and that the bifurcation buckling load is insignificant as a design parameter. The buckling load is determined by use of the Southwell plot as well as by the dynamic method (extrapolation to zero of natural frequency). The agreement between these two methods is good.

- D-19      Spier, E. E., Crippling/Column Buckling Analysis and Test of Graphite-Epoxy-Stiffened Panels, AIAA/ASME/SAE 16th SDM Meeting, Denver, Colorado, May 1975.

*Tests were carried out on three T- and blade-stiffened panels. Buckling and crippling predictions (elastic) based on "elementary methods" were in good agreement with test results. Some of the stress-strain curves for the material (transverse stress and shear) show considerable non-linearity.*

- D-20      Williams, J. G. and Mikulas, Jr., M. M., Analytical and Experimental Study of Structurally Efficient Composite Hat-Stiffened Panels Loaded in Axial Compression, AIAA/ASME/SAE 16th SDM Meeting, Denver, Colorado, May 1975.

*Tests were conducted on a number of hat-stiffened graphite-epoxy panels. The panel dimensions correspond to optimum weight configurations. A number of panels (23) of length 16 in. were critical in local buckling. In the experiments local buckling was observed by use of the Moire pattern and by the observation of strain reversal. These two methods agree with one another but the agreement with theory (BUCLASP) is not good. This is assumed to be due to large deviations from nominal thicknesses and to local initial stresses due to the curing process. The experiments were continued to ultimate failure which typically occurred at about 25% above the experimental buckling load. The authors conclude that "buckled skin concepts" might be possible (but that), the brittle nature of graphite-epoxy composites makes their practicality highly speculative at this time. In addition, six longer panels (60 in.) were tested. These were critical in Euler buckling. The test results in these cases fall between 64 and 90 percent of the critical load. This disagreement appears to be the result of transverse shear effects. The disagreement between experiment and theory reduces a potential 50% weight saving (in comparison to aluminum) to a 32% saving. This estimate does not account for the use of buckled skin concepts for aluminum panels.*

- D-21      Booton, M., Buckling of Imperfect Anisotropic Cylinders Under Combined Loading, Institute for Aerospace Studies, U. of Toronto, UTIAS Rept. No. 203, August 1976.

A literature review is presented on the subject of buckling of cylindrical shells. Available closed form solutions are summarized. A computer program listing is presented for the buckling of anisotropic cylinders under torsion, external pressure and axial load. A number of numerical results indicate that the shapes of the interaction curves vary considerably with the layup. Two fiberglass cylinders were tested. An inside mandrel was used to arrest the buckles and allow repeated testing on the same specimen. The shape of the interaction curves agrees with theoretical results. For comparison with theory an analysis was carried out including axisymmetric imperfections. These were chosen equal to those on the "worst meridian". The test results in axial compression were some 10% below the bifurcation load for the imperfect cylinder. In both cases, the radius to thickness ratio was 234.

- D-22      Spier, E. E., and Klouman, F. L., Postbuckling Behavior of Graphite-Epoxy Laminated Plates and Channels, Army Symposium on Solid Mechanics, Cape Cod, Mass., September 1976.

Graphite-epoxy plates with the unloaded edges simply supported and channel sections were tested in axial compression. Results were compared to results of "classical, closed-form orthotropic elastic buckling theory for flat plates." In many cases, the registered buckling load was well below the theoretical values. For the plates with  $b/t$  varying from 19 to 45, the ratio between experimental and theoretical results varied between 0.61 and 0.88. For the channels the corresponding ratio varied from 0.77 to 1.10. Ultimate failure for the plates varied from 0.71 to 1.55 times the bifurcation buckling loads. For the channels this range was 1.18 to 3.83.

Vestergreen, A Theoretical and Experimental Investigation of the Buckling Behavior of Rectangular CFRP Panels Subjected to Shear Loading The Aeronaut. Res. Inst. of Sweden, FFA Report HU-1818, Stockholm, 1977.

Three panels, 845 x 115 mm, were tested in shear. The layups were 8-ply  $(\pm 45)_S$  ( $t = 1.04$  mm), 10-ply  $(90, 2\pm 45)_S$  ( $t = 1.30$  mm) and 8-ply  $(2 + 90, 2\pm 45)_S$  ( $t = 1.04$  mm). Buckling at test was observed by use of the Moire pattern. The ratios between test and theory for the three specimens were 0.72, 1.27, and 0.75. The tests were continued to failure. The ultimate load values were 2.73, 1.9 and 2.87 times the bifurcation buckling loads (theoretical). The STAGS computer program and Tsai's fracture criterion were used to determine the ultimate load analytically. The agreement between test and theory on this point was surprisingly good, the ratios between experimental and theoretical results being 0.93, 1.16 and 1.07.